16. Permutations

Exercise 16.1

1. Question

Compute

(i) $\frac{30!}{28!}$

(ii) $\frac{11!-10!}{9!}$

(iii). L.C.M. (6!, 7!, 8!)

Answer

i. Given $\frac{30!}{28!}$

$= \frac{30 \times 29 \times 28!}{28!}$

= 30 × 29

= 870

ii. Given $\frac{11!-10!}{9!}$

We know that:

11! = 11 × 10 × 9 × … × 1

10! = 10 × 9 × 8 × … × 1

9! = 9 × 8 × 7 × … × 1

Putting these values, we get,

$\Rightarrow \frac{11!-10!}{9!} = \frac{11 \times 10 \times 9! - 10 \times 9!}{9!}$

$\Rightarrow \frac{11!-10!}{9!} = \frac{9!(110-10)}{9!}$

= 110 - 10

Hence, $\frac{11!-10!}{9!} = 100$

iii. We have to find LCM of 6!, 7!, and 8!

We can write as,

= 8! = 8 × 7 × 6!

= 7! = 7 × 6!

= 6! = 6!

Therefore,

L.C.M (6!, 7!, 8!) = LCM [8 × 7 × 6!, 7 × 6!, 6!]

= 8 × 7 × 6!

Hence, LCM is 8!

2. Question

Prove that $\frac{1}{9!}+\frac{1}{10!}+\frac{1}{11!}=\frac{122}{11!}$

Answer

Given: $\frac{1}{9!}+\frac{1}{10!}+\frac{1}{11!}=\frac{122}{11!}$

L.H.S $=\frac{1}{9!}+\frac{1}{10!}+\frac{1}{11!}$

$=\frac{1}{9!}+\frac{1}{10 \times 9!}+\frac{1}{11 \times 10 \times 9!}$

$=\frac{110 + 11 + 1}{11 \times 10 \times 9!}$

$=\frac{112}{11!}$

Hence, L.H.S = R.H.S

3 A. Question

Find x in each of the following :

$\frac{1}{4!}+\frac{1}{5!}=\frac{x}{6!}$

Answer

We have $\frac{1}{4!}+\frac{1}{5!}=\frac{x}{6!}$

We know that

5! = 5 × 4 × 3 × 2 × 1

6! = 6 × 5 × 4 × 3 × 2 × 1

$\frac{1}{4!}+\frac{1}{5!}=\frac{x}{6!}$

$\Rightarrow \frac{1}{4!}+\frac{1}{5 \times 4!}=\frac{x}{6!}$

$\Rightarrow \frac{5 + 1}{5 \times 4!}=\frac{x}{6!}$

$\Rightarrow \frac{6}{5!}=\frac{x}{6 \times 5!}$

$\Rightarrow x=\frac{6 \times 6 \times 5!}{5!}$

Hence, x = 36

3 B. Question

Find x in each of the following :

$\frac{x}{10!}=\frac{1}{8!}+\frac{1}{9!}$

Answer

We have $\frac{x}{10!} = \frac{1}{8!} + \frac{1}{9!}$

We can write as

10! = 10 × 9!

9! = 9 × 8!

By putting these values

$$\frac{x}{10!} = \frac{1}{8!} + \frac{1}{9 \times 8!}$$

$$\Rightarrow \frac{x}{10!} = \frac{9 + 1}{9 \times 8!}$$

$$\Rightarrow \frac{x}{10!} = \frac{10}{9!}$$

$$\Rightarrow \frac{x}{10 \times 9!} = \frac{10}{9!}$$

$$\Rightarrow x = \frac{10 \times 10 \times 9!}{9!}$$

Hence, x = 100

3 C. Question

Find x in each of the following :

$$\frac{1}{6!} + \frac{1}{7!} = \frac{x}{8!}$$

Answer

We have $\frac{1}{6!} + \frac{1}{7!} = \frac{x}{8!}$

We can write as.

8! = 8 × 7 × 6!

7! = 7 × 6!

Putting the value, we get

$$\frac{1}{6!} + \frac{1}{7!} = \frac{x}{8!}$$

$$\Rightarrow \frac{1}{6!} + \frac{1}{7 \times 6!} = \frac{x}{8!}$$

$$\Rightarrow \frac{1 + 7}{7 \times 6!} = \frac{x}{8!}$$

$$\Rightarrow \frac{8}{7!} = \frac{x}{8!}$$

$$\Rightarrow \frac{8}{7!} = \frac{x}{8 \times 7!}$$

$$\Rightarrow x = \frac{8 \times 8 \times 7!}{7!}$$

Hence, x = 64

4 A. Question

Convert the following products into factorials :

$5 \cdot 6 \cdot 7 \cdot 8 \cdot 9 \cdot 10$

Answer

Given 5 . 6 . 7 . 8 . 9 . 10

Find: Convert into Factorial

Now, We can write as

$= \frac{1\times2\times3\times4\times5\times6\times7\times8\times9\times10}{1\times2\times3\times4}$

Hence, $\frac{10!}{4!}$

4 B. Question

Convert the following products into factorials :

$3 \cdot 6 \cdot 9 \cdot 12 \cdot 15 \cdot 18$

Answer

Given 3 . 6 . 9 . 12 . 15 . 18

Find: Convert into Factorial

$= (3 \times 1) \times (3 \times 2) \times (3 \times 3) \times (3 \times 4) \times (3 \times 5) \times (3 \times 6)$

$= 3^6 (1 \times 2 \times 3 \times 4 \times 5 \times 6)$

$= 3^6 (6!)$

4 C. Question

Convert the following products into factorials :

$(n + 1)(n + 2) (n + 3) \ldots(2n)$

Answer

Given $(n + 1)(n + 2) (n + 3) \ldots(2n)$

Find: Convert into Factorial

$\Rightarrow \frac{(1)(2)(3)\ldots(n)\ldots(n+1)(n+2)(n+3)\ldots 2n}{(1)(2)(3)\ldots(n)}$

$\Rightarrow \frac{(2n)!}{n!}$

4 D. Question

Convert the following products into factorials :

$1 \cdot 3 \cdot 5 \cdot 7 \cdot 9 \ldots(2n - 1)$

Answer

Given $1 \cdot 3 \cdot 5 \cdot 7 \cdot 9 \ldots(2n - 1)$

Find: Convert into Factorial

$\Rightarrow \frac{[(1)(3)(5)\ldots\ldots\ldots(2n-1)][(2)(4)(6)\ldots\ldots(2n)]}{[(2)(4)(6)\ldots(2n)]}$

$\Rightarrow \frac{[(1)(2)(3)(4)\ldots\ldots(2n-1)(2n)]}{2^n[(1)(2)(3)\ldots\ldots\ldots(n)]}$

$= \frac{(2n)!}{2^n n!}$

5 A. Question

Which of the following are true :

(2 + 3)! = 2! + 3!

Answer

Given: (2 + 3)! = 2! + 3!

L.H.S = (2 + 3)!

= 5!

R.H.S = 2! + 3!

= (2 × 1) + (3 × 2 × 1)

= 2 + 6

= 8

Hence, L.H.S ≠ R.H.S

5 B. Question

Which of the following are true :

(2 × 3)! = 2! × 3!

Answer

Given: (2 × 3)! = 2! × 3!

L.H.S = (2 × 3)!

= 6!

= 6 × 5 × 4 × 3 × 2 × 1

= 720

R.H.S = 2! × 3!

= 2 × 1 × 3 × 2 × 1

= 12

Hence, L.H.S ≠ R.H.S

6. Question

Prove that: n! (n + 2) = n! + (n + 1)!

Answer

Given: n! (n + 2) = n! + (n + 1)!

R.H.S. = n! + (n + 1)!

= n! + (n + 1)(n + 1 – 1)!

= n! + (n + 1)n!

= n!(1 + n + 1)

= n! (n + 2) = L.H.S

L.H.S = R.H.S

Hence, Proved.

7. Question

If (n + 2)! = 60 [(n – 1)!], find n.

Answer

Given (n + 2)! = 60 [(n – 1)!]

To Find : n

⇒ (n + 2) × (n + 1) × (n) × (n-1)! = 60 [(n – 1)!]

⇒(n + 2) × (n + 1) × (n) = 60

⇒ (n + 2) × (n + 1) × (n) = 5 × 4 × 3

n = 3

Hence, n = 3

8. Question

If (n + 1)! = 90 [(n – 1)!], find n.

Answer

Given: (n + 1)! = 90 [(n – 1)!]

To Find : n ?

⇒ (n + 1) × (n) × (n-1)! = 60 [(n – 1)!]

⇒ (n + 1) × (n) = 90

⇒ (n + 1) × (n) = 10 × 9

n = 9

Hence, n = 9

9. Question

If (n + 3)! = 56 [(n + 1)!], find n.

Answer

Given: (n + 3)! = 56 [(n + 1)!]

To Find : n

⇒ (n + 3) × (n + 2) × (n + 1)! = 56 [(n + 1)!]

⇒ (n + 3) × (n + 2) = 56

⇒ (n + 3) × (n + 2) = 8 × 7

⇒n + 3 = 8

n = 5

Hence, n = 5

10. Question

If $\frac{(2n)!}{3!(2n-3)!}$ and $\frac{n!}{2!(n-2)!}$ are in the ratio 44 : 3, find n.

Answer

Given: $\frac{(2n)!}{3!(2n-3)!} : \frac{n!}{2!(n-2)!} = 44 : 3$

Find: n

$$\Rightarrow \frac{(2n)!}{3!\,(2n-3)!} : \frac{n!}{2!\,(n-2)!} = 44 : 3$$

$$\Rightarrow \frac{(2n)!}{3!\,(2n-3)!} \times \frac{2!\,(n-2)!}{n!} = \frac{44}{3}$$

$$\Rightarrow \frac{(2n)(2n-1)(2n-2)(2n-3)!}{3 \times 2!\,(2n-3)!} \times \frac{2!\,(n-2)!}{n(n-1)(n-2)!} = \frac{44}{3}$$

$$\Rightarrow \frac{(2n)(2n-1)(2n-2)}{3} \times \frac{1}{n(n-1)} = \frac{44}{3}$$

$$\Rightarrow \frac{4(2n-1)n(n-1)}{3} \times \frac{1}{n(n-1)} = \frac{44}{3}$$

$$\Rightarrow \frac{4(2n-1)}{3} = \frac{44}{3}$$

⇒ (2n - 1) = 11

⇒2n = 12

n = 6

Hence, n = 6

11 A. Question

Prove that :

$$\frac{n!}{(n-r)!} = n(n-1)(n-2)\ldots(n(r-1))$$

Answer

Given: $\frac{n!}{(n-r)!} = n(n-1)(n-2)\ldots(n(r-1))$

L.H.S $= \frac{n!}{(n-r)}$

$$\Rightarrow \frac{n!}{(n-r)} = \frac{n(n-1)(n-2)(n-3)\ldots\ldots\ldots(n-r+1)(n-r)!}{(n-r)!}$$

$$\Rightarrow \frac{n!}{(n-r)} = n(n-1)(n-2)(n-3)\ldots\ldots\ldots(n-(r+1))$$

⇒ $\frac{n!}{(n-r)} = n(n-1)(n-2)(n-3)\ldots\ldots\ldots(n-r-1)$ R.H.S

L.H.S = R.H.S

Hence, Proved.

11 B. Question

Prove that :

$$\frac{n!}{(n-r)!r!} + \frac{n!}{(n-r+1)!(r-1)!} = \frac{(n+1)!}{r!(n-r+1)!}$$

Answer

Given: $\frac{n!}{(n-r)!\,r!} + \frac{n!}{(n-r+1)!(r-1)!} = \frac{(n+1)!}{r!(n-r+1)!}$

L.H.S $= \frac{n!}{(n-r)!\,r!} + \frac{n!}{(n-r+1)!(r-1)!}$

$$\Rightarrow \frac{n!}{(n-r)!\,r!} + \frac{n!}{(n-r+1)!\,(r-1)!}$$
$$= \frac{n!}{(n-r)!\,r!} + \frac{n!}{(n-r+1)!\,(r-1)!}$$

$$\Rightarrow \frac{n!}{(n-r)!\,r!} + \frac{n!}{(n-r+1)!\,(r-1)!} = \frac{n!\,(n-r+1) + n!\,r!}{r!\,(n-r+1)[(n-r)!]}$$

$$\Rightarrow \frac{n!}{(n-r)!\,r!} + \frac{n!}{(n-r+1)!\,(r-1)!} = \frac{n!\,(n+1) - n!\,r! + n!\,r!}{r!\,(n-r+1)(n-r)!}$$

$$\Rightarrow \frac{n!}{(n-r)!\,r!} + \frac{n!}{(n-r+1)!\,(r-1)!} = \frac{n!\,(n+1)}{r!\,(n-r+1)(n-r)}$$

$= \frac{(n+1)!}{r!(n-r+1)!}$ R.H.S

L.H.S = R.H.S

Hence, Proved.

12. Question

Prove that : $\frac{(2n+1)!}{n!} = 2^n\{1.3.5...(2n-1)(2n+1)\}$

Answer

Given: $\frac{(2n+1)!}{n!} = 2^n\{1.3.5\ldots(2n-1)(2n+1)\}$

L.H.S $= \frac{(2n+1)!}{n!}$

$$\Rightarrow \frac{(2n+1)!}{n!} = \frac{(2n+1)(2n)(2n-1)\ldots(4)(3)(2)(1)}{n!}$$

$$\Rightarrow \frac{(2n+1)!}{n!}$$
$$= \frac{[(1)(3)(5)\ldots\ldots\ldots\ldots(2n-1)(2n+1)][(2)(4)(6)\ldots\ldots\ldots(2n)]}{n!}$$

$$\Rightarrow \frac{(2n+1)!}{n!}$$
$$= \frac{2^n[(1)(3)(5)\ldots\ldots\ldots\ldots(2n-1)(2n+1)][(1)(2)(3)\ldots\ldots\ldots(n)]}{n!}$$

$$\Rightarrow \frac{(2n+1)!}{n!} = \frac{2^n[(1)(3)(5)\ldots\ldots\ldots\ldots(2n-1)(2n+1)]n!}{n!}$$

$= 2^n\{1.3.5\ldots(2n-1)(2n+1)\}$ R.H.S

L.H.S = R.H.S

Hence, Proved.

Exercise 16.2

1. Question

In a class, there are 27 boys and 14 girls. The teacher wants to select 1 boy and 1 girl to represent the class in a function. In how many ways can the teacher make this selection?

Answer

Given: 27 boys and 14 girls

We have to select one boy from 27 boys and one girl from 14 girls.

Therefore, a number of ways to select one boy are ${}^{27}C_1$ and similarly the number of ways to select one girl is ${}^{14}C_1$.

Hence the number of ways to select 1 boy and 1 girl to represent the class in a function is ${}^{14}C_1 \times {}^{27}C_1 = 14 \times 27 = 378$

2. Question

A person wants to buy one fountain pen, one ball pen, and one pencil from a stationery shop. If there are 10 fountain pen varieties, 12 ball pen varieties and 5 pencil varieties, in how many ways can he select these articles?

Answer

Given: 10 fountain pens, 12 ball pens, and 5 pencil

We have to select one fountain pen from 10 fountain pens, one ball pen from 12 ball pens and one pencil from 5 pencils.

Therefore, the number of ways to select one fountain pen is ${}^{10}C_1$ and similarly the number of ways to select one ball pen is ${}^{12}C_1$ and number of ways to select one pencil from 5 pencils in ${}^{5}C_1$

Hence, the number of ways to select one fountain pen, one ball pen and one pencil from a stationery shop is ${}^{10}C_1 \times {}^{12}C_1 \times {}^{5}C_1 = 10 \times 12 \times 5 = 600$

3. Question

From Goa to Bombay there are two roots; air, and sea. From Bombay to Delhi there are three routes; air, rail, and road. From Goa to Delhi via Bombay, how many kinds of routes are there?

Answer

we have to go from Goa to Delhi via Bombay.

Given: the number of ways from goa to Mumbai is air and sea

Therefore, the number of ways to go from Goa to Mumbai is ${}^{2}C_1$

Given a number of ways from Mumbai to Delhi are air, rail, and road.

Therefore, the number of ways to go from Mumbai to Delhi is ${}^{3}C_1$

Hence the number of ways to go from Goa to Delhi via Bombay is ${}^{2}C_1 \times {}^{3}C_1 = 2 \times 3 = 6$

4. Question

A mint prepares metallic calendars specifying months, dates and days in the form of monthly sheets (one plate for each month). How many types of calendars should it prepare to serve for all the possibilities in future years?

Answer

According to the given question, there can be four types of month possible. Months with 30 or 31 days for normal months and for February it can be 28 or 29 days

Now for the months, it can start from any of one day of the week, so there are 7 possibilities.

Hence, the number of types calendars should it prepare to serve for all the possibilities in future years is $7 \times 4 = 28$

5. Question

There are four parcels and five post-offices. In how many different ways can the parcels be sent by registered post?

Answer

According to the given question one parcel can be posted in 5 ways, that is in either of the one post offices so $^5C_1$. Similarly, for other parcels also it can be posted in $^5C_1$ ways.

Hence the number of ways the parcels be sent by registered post is $^5C_1 \times {}^5C_1 \times {}^5C_1 \times {}^5C_1 = 5 \times 5 \times 5 \times 5 =$ 625

6. Question

A coin is tossed five times, and outcomes are recorded. How many possible outcomes are there?

Answer

Given: A coin is tossed 5 times, so each time the outcome is either heads or tails, which implies two possibilities are possible.

So, total possible outcomes possible are $^2C_1 \times {}^2C_1 \times {}^2C_1 \times {}^2C_1 \times {}^2C_1 = 2 \times 2 \times 2 \times 2 \times 2 = 32$

7. Question

In how many ways can an examinee answer a set of ten true/false type questions?

Answer

Given: An examinee can answer a question either true or false, which implies two possibilities are possible.

So total possible outcomes possible are $^2C_1 \times {}^2C_1 \times {}^2C_1 \times {}^2C_1 \times {}^2C_1 \times {}^2C_1 \times {}^2C_1 \times {}^2C_1 \times {}^2C_1 \times {}^2C_1 = 2^{10}$

8. Question

A letter lock consists of three rings each marked with 10 different letters. In how many ways it is possible to make an unsuccessful attempt to open the lock?

Answer

Given: Each ring consists of 10 letters and there are three rings. Only one code is correct which will open the lock.

Therefore one subtracted from total possible outcomes will give the number of unsuccessful attempts.

At a time only one letter can appear in each ring

So total number of possible outcomes are $^{10}C_1 \times {}^{10}C_1 \times {}^{10}C_1 = 1000$

Hence, the number of unsuccessful attempts to open the lock is 1000 - 1 = 999

9. Question

There are 6 multiple choice questions in an examination. How many sequences of answers are possible, if the first three questions have 4 choices each and the next three have 2 each?

Answer

Given: Multiple choice question, only one answer is correct of the given options.

So, for the first three questions only one answer is correct out of four, similarly, for the remaining three question, only one answer is correct out of two.

Number of outcomes possible is $^4C_1 \times {}^4C_1 \times {}^4C_1$ for the first three and $^2C_1 \times {}^2C_1 \times {}^2C_1$ for the remaining three.

Hence, total possible outcomes possible are $^4C_1 \times {}^4C_1 \times {}^4C_1 \times {}^2C_1 \times {}^2C_1 \times {}^2C_1 = 512$

10. Question

There are 5 books on Mathematics and 6 books on Physics in a book shop. In how many ways can a student buy:

(i) a Mathematics book and a Physics book

(ii) either a Mathematics book or a Physics book?

Answer

(i) Given there are 5 books of mathematics and 6 books of physics. In order to buy one mathematics book number of ways are ${}^5C_1$ similarly to buy one physics book number of ways are ${}^6C_1$

Hence, the number of ways a student buy a Mathematics book and a Physics book is ${}^5C_1 \times {}^6C_1 = 5 \times 6 = 30$

(ii) Given there is a total of 11 books, so in order to buy either a Mathematics book or a Physics book it means that only one book out of eleven is bought.

Hence, the number of ways in which a student can either buy either a Mathematics book or a Physics book is ${}^{11}C_1 = 11$

11. Question

Given 7 flags of different colors, how many different signals can be generated if a signal requires the use of two flags, one below the other?

Answer

Given: seven flags are available and out of which two are needed to make a signal.

From this, we can say, that we have to select two flags out of seven and arrange these two flags to get one signal.

So, number of ways to select two flags out of seven is ${}^7C_2$. These flags can be arranged in 2! ways one below the other

Hence total number of signals possible are ${}^7C_2 \times 2! = 42$

123. Question

A team consists of 6 boys and 4 girls, and other has 5 boys and 3 girls. How many single matches can be arranged between the two teams when a boy plays against a boy, and a girl plays against a girl?

Answer

Given: Singles matches are to be played, either a boy plays against a boy, and a girl plays against a girl.

A number of ways to select a boy from team 1 is ${}^6C_1$. Similarly, Number of ways to select a boy from team 2 is ${}^5C_1$

Hence number of singles matches between boys is ${}^6C_1 \times {}^5C_1 = 6 \times 5 = 30$

A number of ways to select a girl from team 1 is ${}^4C_1$. Similarly, Number of ways to select a girl from team 2 is ${}^3C_1$

Hence number of singles matches between girls is ${}^3C_1 \times {}^4C_1 = 4 \times 3 = 12$

The total number of matches = 30 + 12 = 42.

13. Question

Twelve students compete in a race. In how many ways first three prizes be given?

Answer

Given: A total of three prizes are to be given.

Number of ways to select the winner of the first prize is ${}^{12}C_1$

Number of ways to select the winner of the second prize is ${}^{11}C_1$ (11 since one student is already given a prize)

Number of ways to select the winner of the third prize is ${}^{10}C_1$ (10 since two students are already given a prize)

Hence, total number of ways is ${}^{12}C_1 \times {}^{11}C_1 \times {}^{10}C_1 = 12 \times 11 \times 10 = 1320$

14. Question

How many A.P.'s with 10 terms are there whose first term is in the set {1, 2, 3} and whose common difference is in the set {1, 2, 3, 4, 5}?

Answer

Each AP consists of a unique first term and common difference.

Number of ways to select the first term of a given set is ${}^{3}C_1 = 3$

Number of ways to select a common difference of given set is ${}^{5}C_1 = 5$

Hence total number of AP's possible are ${}^{3}C_1 \times {}^{5}C_1 = 3 \times 5 = 15$

15. Question

From among the 36 teachers in a college, one principal, one vice-principal and the teacher-in-charge are to be appointed. In how many ways can this be done?

Answer

Given: A total of three positions are to be given.

Number of ways to select principal is ${}^{36}C_1$

Number of ways to select vice-principal is ${}^{35}C_1$ (35 since one position is already given)

Number of ways to select teacher in charge is ${}^{34}C_1$ (34 since two positions are already given)

Hence, total number of ways is ${}^{36}C_1 \times {}^{35}C_1 \times {}^{34}C_1 = 36 \times 35 \times 34 = 42840$

16. Question

How many three-digit numbers are there with no digit repeated

Answer

Given: the three-digit number is required without digit repetition

Assume we have three boxes; the first box can be filled with any one of the nine digits (0 not allowed at first place).

Therefore, possibilities are ${}^{9}C_1$, the second box can be filled with any one of the nine digits available possibilities are ${}^{9}C_1$, the third box can be filled with any one of the eight digits available possibilities are ${}^{8}C_1$.

Hence, number of total outcomes possible are ${}^{9}C_1 \times {}^{9}C_1 \times {}^{8}C_1 = 9 \times 9 \times 8 = 648$

17. Question

How many three-digit numbers are there?

Answer

Given: The three-digit number is required

Assume we have three boxes, first box can be filled with any one of the nine digits (zero not allowed at first position) therefore possibilities are ${}^{9}C_1$, the second box can be filled with any one of the ten digits available possibilities are ${}^{10}C_1$, third box can be filled with any one of the ten digits available possibilities are ${}^{10}C_1$.

Hence number of total outcomes possible are ${}^{9}C_1 \times {}^{10}C_1 \times {}^{10}C_1 = 9 \times 10 \times 10 = 900$

18. Question

How many three-digit odd numbers are there?

Answer

In odd numbers, the last digit consists of (1, 3, 5, 7, 9)

Assume we have three boxes, first box can be filled with any one of the nine digits (zero not allowed at first position) therefore possibilities are $^9C_1$, the second box can be filled with any one of the ten digits available possibilities are $^{10}C_1$, third box can be filled with any one of the five digits (1,3,5,7,9) $^5C_1$

Hence, number of total outcomes possible are $^9C_1 \times {}^{10}C_1 \times {}^5C_1 = 9 \times 10 \times 5 = 450$

19. Question

How many different five-digit number license plates can be made if

i. the first digit cannot be zero, and the repetition of digits is not allowed,

ii. the first-digit cannot be zero, but the repetition of digits is allowed?

Answer

(i) Given: Five-digit number is required in which the first digit cannot be zero, and the repetition of digits is not allowed.

Assume five boxes, now the first box can be filled with one of the nine available digits, so the possibility is $^9C_1$

Similarly, the second box can be filled with one of the nine available digits, so the possibility is $^9C_1$

the third box can be filled with one of the eight available digits, so the possibility is $^8C_1$

the fourth box can be filled with one of the seven available digits, so the possibility is $^7C_1$

the fifth box can be filled with one of the six available digits, so the possibility is $^6C_1$

Hence, the number of total possible outcomes is $^9C_1 \times {}^9C_1 \times {}^8C_1 \times {}^7C_1 \times {}^6C_1 = 9 \times 9 \times 8 \times 7 \times 6 = 27216$

(ii) Given: Five-digit number is required in which the first digit cannot be zero, and the repetition of digits is allowed

assume five boxes, now the first box can be filled with one of the nine available digits, so the possibility is $^9C_1$

Similarly, the second box can be filled with one of the ten available digits, so the possibility is $^{10}C_1$

the third box can be filled with one of the ten available digits, so the possibility is $^{10}C_1$

the fourth box can be filled with one of the ten available digits, so the possibility is $^{10}C_1$

the fifth box can be filled with one of the ten available digits, so the possibility is $^{10}C_1$

Hence, the number of total possible outcomes is $^9C_1 \times {}^{10}C_1 \times {}^{10}C_1 \times {}^{10}C_1 \times {}^{10}C_1 = 9 \times 10 \times 10 \times 10 \times 10 = 90000$

20. Question

How many four-digit numbers can be formed with the digits 3, 5, 7, 8, 9 which are greater than 7000, if repetition of digits is not allowed?

Answer

Given: Four-digit number is required which is greater than 7000

Assume four boxes, now in the first box can either be one of the three numbers 7, 8 or 9, so there are three possibilities which are $^3C_1$

In the second box, the numbers can be any of the four digits left, so the possibility is $^4C_1$

Similarly, for the third box, the numbers can be any of the three digits left, so the possibility is $^3C_1$

for the fourth box, the numbers can be any of the two digits left, so the possibility is ${}^{2}C_{1}$

Hence the total number of possible outcomes is ${}^{3}C_{1} \times {}^{4}C_{1} \times {}^{3}C_{1} \times {}^{2}C_{1} = 3 \times 4 \times 3 \times 2 = 72$

21. Question

How many four-digit numbers can be formed with the digits 3, 5, 7, 8, 9 which are greater than 8000, if repetition of digits is not allowed?

Answer

Given: Four-digit number is required which is greater than 8000

Assume four boxes, now in the first box can either be one of the two numbers 8 or 9, so there are two possibilities which is ${}^{2}C_{1}$

In the second box, the numbers can be any of the four digits left, so the possibility is ${}^{4}C_{1}$

Similarly, for the third box, the numbers can be any of the three digits left, so the possibility is ${}^{3}C_{1}$

for the fourth box, the numbers can be any of the two digits left, so the possibility is ${}^{2}C_{1}$

Hence total number of possible outcomes is ${}^{2}C_{1} \times {}^{4}C_{1} \times {}^{3}C_{1} \times {}^{2}C_{1} = 2 \times 4 \times 3 \times 2 = 48$

22. Question

In how many ways can six persons be seated in a row?

Answer

Given: Six persons are to be arranged in a row

Assume six seats, now in the first seat, any one of six members can be seated, so the total number of possibilities is ${}^{6}C_{1}$

Similarly, in the second seat, any one of five members can be seated, so the total number of possibilities is ${}^{5}C_{1}$

In the third seat, any one of four members can be seated, so the total number of possibilities is ${}^{4}C_{1}$

In the fourth seat, any one of three members can be seated, so the total number of possibilities is ${}^{3}C_{1}$

In the fifth seat, any one of two members can be seated, so the total number of possibilities is ${}^{2}C_{1}$

In the sixth seat, only one remaining person can be seated, so the total number of possibilities is ${}^{1}C_{1}$

Hence the total number of possible outcomes $= {}^{6}C_{1} \times {}^{5}C_{1} \times {}^{4}C_{1} \times {}^{3}C_{1} \times {}^{2}C_{1} \times {}^{1}C_{1} = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$

23. Question

How many 9-digit numbers of different digits can be formed

Answer

Given: Nine-digit number is required in which the first digit cannot be zero and the repetition of digits is not allowed.

2	3	4	5	6	7	8	9	9

Assume nine boxes, now the first box can be filled with one of the nine available digits, so the possibility is ${}^{9}C_{1}$

Similarly, the second box can be filled with one of the nine available digits, so the possibility is ${}^{9}C_{1}$

the third box can be filled with one of the eight available digits, so the possibility is ${}^{8}C_{1}$

the fourth box can be filled with one of the seven available digits, so the possibility is ${}^{7}C_1$

the fifth box can be filled with one of the six available digits, so the possibility is ${}^{6}C_1$

the sixth box can be filled with one of the six available digits, so the possibility is ${}^{5}C_1$

the seventh box can be filled with one of the six available digits, so the possibility is ${}^{4}C_1$

the eighth box can be filled with one of the six available digits, so the possibility is ${}^{3}C_1$

the ninth box can be filled with one of the six available digits, so the possibility is ${}^{2}C_1$

Hence the number of total possible outcomes is ${}^{9}C_1 \times {}^{9}C_1 \times {}^{8}C_1 \times {}^{7}C_1 \times {}^{6}C_1 = 9 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 = 9\,(9!)$

24. Question

How many odd numbers less than 1000 can be formed by using the digits 0, 3, 5, 7 when repetition of digits is not allowed?

Answer

Given: Odd number less than 1000 is required.

In order to make the number odd, the last digit has to either of (3, 5, 7)

(No Zero)	(Any 3 digits left)	(3, 5, or 7)

Assume three boxes, in the first either of the three digits (3,5,7) can be placed, so the possibility is ${}^{3}C_1$

Case 1: Middle digit is zero

If the middle digit is zero, number of ways of placing odd numbers on the second box = 2

Hence, the total number of ways = 3 × 2 = 6 ways

Case 2: Middle digit is an odd number

Number of ways of filling middle box = 2

Number of ways of filling third box = 1

Hence, the total number of ways = 3 × 3 = 9 ways

Hence total number of outcomes possible = 6 + 9 = 15 ways

25. Question

How many 3 - digit numbers are there, with distinct digits, with each digit odd?

Answer

In the question, it is given that we have to find three - digit numbers with distinct digits which means the digits should be nonrepeating and all the digits should be odd means no even digit.

Numbers by which we form the three digit numbers are 1, 3, 5, 7, 9 only the odd ones.

We will use the concept of multiplication because we have three sub jobs and each job is dependent on the other because a number selected on hundred’s place will not appear in ones and tens place.

The number of ways in which we can form three - digit numbers with odd digits is , 5 × 4 × 3 = 60.

3	4	5

We can also do it by 3 × 4 × 5 = 60 , there are total of 5 choices in the first digit on any place then it becomes 4 in the second digit place because some number has been placed in the first digit out of 5 , so out of rest 4 numbers one number is again consumed at second place so at the end 3 numbers are left to fit the last and third place of our 3 digit number.

26. Question

How many different numbers of six digits each can be formed from the digits 4, 5, 6, 7, 8, 9 when repetition of digits is not allowed?

Answer

In the question, we have to find the possible number of 6 digit numbers formed by the numbers 4, 5, 6, 7, 8, 9 when repetition of digits is not allowed.

We will use the concept of multiplication because there are six sub jobs dependent on each other because a number appearing on any one place will not appear in any other place.

The number of ways in which we can form six digit numbers with the help of given numbers is $6 \times 5 \times 4 \times 3 \times 2 \times 1 = 6! = 720$

The numbers occurring on first place from the left have 6 choices and when one number is placed then number occurring on the second place from the left will have 5 choices and so on one fewer choice will be available to every next place till one occurs.

27. Question

How many different numbers of six digits can be formed from the digits 3, 1, 7, 0, 9, 5 when the repetition of digits is not allowed?

Answer

In the question, we have to find the possible number of 6 digit numbers formed by the numbers 0, 1, 3, 5, 7, 9 when repetition of digits is not allowed.

We will use the concept of multiplication because there are six sub jobs dependent on each other because a number appearing on any one place will not appear in any other place.

Since zero cannot be used in the first position from the left because then it will become a 5 digit number , so we have total of 6 numbers to choose from, but we exclude zero , so we have 5 numbers to choose for the first place , and out of 5 only one number gets occupied , so 4 numbers are left but we had only 5 choices out of 6 and zero can come on the second position from the left side , so remaining choices for the second position are 5 , the number of choices will decrease by one as we keep on going right side.

The number of ways in which we can form six digit numbers with the help of given numbers is $5 \times 5 \times 4 \times 3 \times 2 \times 1 = 5 \times 5! = 120 \times 5 = 600$

The numbers occurring on first place from left have 5 choices because zero cannot come here and when one number is placed then number occurring on second place from left should have 4 choices but now zero is included in our choice because of the second position , so there will be 5 choice instead of 4 , and so on one fewer choice will be available to every next place until one occurs.

28. Question

How many four digit different numbers, greater than 5000 can be formed with the digits 1, 2, 5, 9, 0 when repetition of digits is not allowed?

Answer

In the question, we have to find the possible number of 4 digit numbers greater than 5000 formed by the numbers 0, 1, 2, 5, 9 when repetition of digits is not allowed.

We will use the concept of multiplication because there are four sub jobs dependent on each other because a number appearing on any one place will not appear in any other place.

For the first place from left we have two choices 5 and 9 because only then our number will be greater than 5000 , for the second place we have 4 choices because out of two one is assigned to the first position from left and total choice of numbers are 5 , so $5 - 1 = 4$, the number of choices will decrease by one as we keep on going right side.

The number of ways in which we can form six digit numbers with the help of given numbers is $2 \times 4 \times 3 \times 2 = 2 \times 4! = 2 \times 24 = 48$

29. Question

Serial numbers for an item produced in a factory are to be made using two letters followed by four digits (0 to 9). If the letters are to be taken from six letters of English alphabet without repetition and the digits are also not repeated in a serial number, how many serial numbers are possible?

Answer

In the question, we have to find the possible number of 4 digits and 2 alphabetical place serial numbers formed by the numbers 0 to 9 and with only 6 English alphabets when repetition of digits and alphabets is not allowed.

We will use the concept of multiplication because there are six sub jobs dependent on each other because a number and alphabet appearing on any one place will not appear in any other place.

In first position from left we have 6 alphabet choices, so in the second position, we will have 5 choices because no repetition is allowed. On the first position from left for digits or third position from the left of serial number we will have 10 choices, and for next position, we will have 9 choices, the number of choices will decrease by one as we keep on going right side. Because on serial numbers we can write zero on the left most position of numbers.

The number of ways in which we can form four digit with two alphabets serial numbers with the help of given data is $6 \times 5 \times 10 \times 9 \times 8 \times 7 = 151200$

Note the positions of the two alphabets is fixed that is the first two and then four digits are placed afterward if the positions were not fixed then the answer would vary.

30. Question

A number lock on a suitcase has 3 wheels each labeled with ten digits 0 to 9. If the opening of the lock is a particular sequence of three digits with no repeats, how many such sequences will be possible? Also, find the number of unsuccessful attempts to open the lock.

Answer

In the question, we have to find the possible number of 3 digit numbers formed by the numbers 0 to 9 when repetition of digits is not allowed.

We will use the concept of multiplication because there are three sub jobs dependent on each other because a number appearing on any one place will not appear in any other place.

In the first position from left we will have ten choices, in the second position we will have nine choices, and in the third position, we will have eight choices because repetition is not allowed and one digit is occupied in each position.

The number of ways in which we can form three - digit numbers with the help of given data is $10 \times 9 \times 8 = 720$

There will be only one correct combination out of these so the incorrect combinations will be $720 - 1 = 719$

31. Question

A customer forgets a four - digit code for an Automatic Teller Machine (ATM) in a bank. However, he remembers that this code consists of digits 3, 5, 6 and 9. Find the largest possible number of trails necessary to obtain the correct code.

Answer

In the question, we have to find the possible number of 4 digit numbers formed by the numbers 3, 5, 6, 9 when repetition of digits is not allowed.

We will use the concept of multiplication because there are four sub jobs dependent on each other because a number appearing on any one place will not appear in any other place because ***all the four numbers are to be used as they are in the code***.

The first position from the left will have four choices; the second position will have three choices because one number is consumed in the first position , the number of choices will decrease by one as we keep on going right side.

The number of ways in which we can form four-digit numbers with the help of given data is $4 \times 3 \times 2 \times 1 = 24$

32. Question

In how many ways can get three jobs I, II and III be assigned to three persons A, B and C if one person is assigned only one job and all are capable of doing each job?

Answer

In the question, we have to find the possible number of ways in which three jobs I, II, III can be distributed among three guys A, B, C, and one job can be assigned to one person only.

We will use the concept of multiplication because there are three sub jobs dependent on each other because a job assigned to one person cannot be assigned to another person.

The job I has three choices to be assigned and when it is assigned to one person , there are two persons left, and Job II can be assigned to only two persons after assigning Job II , Job III has only one choice because there is only one person left.

The number of ways in which we can distribute three jobs between three persons with the help of given data is $3 \times 2 \times 1 = 6$.

33. Question

How many four digit natural numbers are not exceeding 4321 can be formed with the digits 1, 2, 3 and 3 if the digits can repeat?

Answer

We have to find the possible number of four digit numbers which are less than 4321 and are formed with the numbers 1, 2, 3, 4 when repetition of digits is allowed.

We will use the concept of multiplication because there are four sub jobs dependent on each other because the number 4321 cannot be exceeded and we have to look at repetitions carefully.

First, let us consider all the cases in which we do not assign 4 on the thousand's place which means we will only assign 1, 2, 3 only and repetition in all other places.

The number of ways in which we can assign values to four digit number, but the thousand's place will not be assigned 4 when repetition is allowed with the help of given data is $3 \times 4 \times 4 \times 4 = 192$.

Now we will consider the case when at thousand's place there is only 4 and nothing else.

So there will be only one choice on thousand's place that is 4 , there will be two choices on hundred's place because there will not be 3 and 4 because then our number will become greater than 4321 and 3 will be dealt later by fixing it in hundred's place like 4 is fixed in this case. The rest places have all the choices.

The number of ways in which we can assign values to the four-digit number and the thousand's place is only assigned 4 but hundred's place is not assigned 3 when repetition is allowed with the help of given data is $1 \times 2 \times 4 \times 4 = 32$.

Now we will consider the case when thousand's place is fixed by 4 and hundred's place is fixed by 3, ten's place is fixed by 1.

There is one choice each on thousand's and hundred's place, but on ten's place also there is one choice which is 1 , so a number of choices on one's place will be four.

The number of ways in which we can assign values to four-digit number and the thousand's place is only assigned 4, hundred's place is only assigned 3, ten's place is only assigned 1 when repetition is allowed with the help of given data is $1 \times 1 \times 1 \times 4 = 4$.

Now we will consider the case when thousand's place is fixed by 4 and hundred's place is fixed by 3, ten's place is fixed by 2.

There is one choice each on thousand's and hundred's place, but on ten's place also there is one choice which is 2 , so a number of choices on one's place will be one.

So the number coming to my mind will be 4321 , which is one number.

Hence total numbers occurring are $192+32+4+1 = 229$.

34. Question

How many numbers of six digits can be formed from the digits 0, 1, 3, 5, 7 and 9 when no digit is repeated? How many of them are divisible by 10?

Answer

In the question, we have to find the possible number of 6 digit numbers formed by the numbers 0, 1, 3, 5, 7, 9 when repetition of digits is not allowed.

We will use the concept of multiplication because there are six sub jobs dependent on each other because a number appearing on any one place will not appear in any other place.

The first position from left will have five choices because zero cannot be assigned to that position because then our number will become a five digit number instead of six , the second position will also have five choices because when a number is occupied by the first position then four numbers are left but we ignored zero for the first position and for the second position we can use zero, the number of choices will decrease by one as we keep on going right side.

The number of ways in which we can form six digit numbers with the help of given data is $5 \times 5 \times 4 \times 3 \times 2 \times 1 = 600$

Numbers which are divisible by 10 should have zero at their one’s place , so we will fix zero on one’s place, and the rest of the positions will have choices accordingly.

The number of ways in which we can form six digit numbers when zero is fixed in one’s place and which are divisible by 10 is $5 \times 4 \times 3 \times 2 \times 1 \times 1 = 5! = 120$

35. Question

If three six faced die each marked with numbers 1 to 6 on six faces, are thrown find the total number of possible outcomes.

Answer

We have to find the number of total outcomes when three dices are rolled the outcomes can be different and can be same.

We will use the concept of multiplication because there are three sub jobs dependent on each other and one job is performed one after the other.

The choices of outcomes from one dice are six; there are three dices so choices for each dice will be six.

The number of outcomes which are formed when three dices are rolled one after the other are $6 \times 6 \times 6 = 6^3 = 216$

36. Question

A coin is tossed three times, and the outcomes are recorded. How many possible outcomes are there? How many possible outcomes if the coin is tossed four times? Five times? N times?

Answer

We have to find the number of total outcomes when a coin is tossed three times, the outcomes can be different and can be the same.

We will use the concept of multiplication because there are three sub jobs dependent on each other and one job is performed one after the other.

The choices of outcomes from one coin when flipped once are two; the coin is flipped three times, so each time the number of outcomes will be two.

The number of outcomes which are formed when a coin is flipped thrice, one after the other, are $2 \times 2 \times 2 = 2^3 = 8$

The number of outcomes which are formed when a coin is flipped four times, one after the other, are $2 \times 2 \times 2 \times 2 = 2^4 = 16$

The number of outcomes which are formed when a coin is flipped five times, one after the other, are $2 \times 2 \times 2 \times 2 \times 2 = 2^5 = 32$

The number of outcomes which are formed when a coin is flipped n times, one after the other, are $2 \times 2 \times 2$............till n times...... $\times 2 = 2^n$

37. Question

How many numbers of four-digit can be formed with the digits 1, 2, 3, 4, 5 if the digit can be repeated in the same number?

Answer

We have to find the possible number of four digit numbers that are formed with the numbers 1, 2, 3, 4, 5 when repetition of digits is allowed.

We will use the concept of multiplication because there are four sub jobs dependent on each other and are performed one after the other.

There is a total of five choices for every position as repetition is allowed and there is no zero in the given choices of numbers.

The number of ways in which we can form four digit numbers when repetition of digits is allowed along with given numbers $5 \times 5 \times 5 \times 5 = 5^4 = 625$

38. Question

How many can three digit number be formed by using the digits 0, 1, 3, 5, 7 while each digit may be repeated any number of times?

Answer

We have to find the possible number of three-digit numbers that are formed with the numbers 0, 1, 3, 5, 7 when repetition of digits is allowed.

We will use the concept of multiplication because there are four sub jobs dependent on each other and are performed one after the other.

There are four choices for hundred's position because there is zero also which cannot be used in hundred's place because then our number will become a two digit number instead of a three digit number, there are five choices for the ten's place because there are a total of five numbers, hundred's placed in which zero was not included but in ten's place zero is included and in one's place there are also five choices because repetition is allowed.

The number of ways in which we can form three digit numbers when repetition of digits is allowed along with given numbers $4 \times 5 \times 5 = 100$

39. Question

How many natural numbers less than 1000 can be formed from the digits 0, 1, 2, 3, 4, 5 when a digit may be repeated any number of times?

Answer

We have to find the possible number of numbers that are formed with the numbers 0, 1, 2, 3, 4, 5 which are less than 1000 when repetition of digits is allowed.

We will use the concept of multiplication because there are sub jobs dependent on each other and are performed one after the other.

First we will make three-digit numbers, There are five choices for hundred's position because there is zero also which cannot be used in hundred's place because then our number will become a two digit number instead of a three digit number, there are six choices for the ten's place because there are a total of six numbers, hundred's placed in which zero was not included but in ten's place zero is included and in one's place there are also six choices because repetition is allowed.

The number of ways in which we can form three digit numbers when repetition of digits is allowed along with given numbers $5 \times 6 \times 6 = 180$

Secondly we will make two-digit numbers, There are five choices for ten's position because there is zero also which cannot be used in ten's place because then our number will become a one digit number instead of a two digit number, there are six choices for the one's place because there are a total of six numbers, ten place in which zero was not included but in one's place zero is included.

The number of ways in which we can form two digit numbers when repetition of digits is allowed along with given numbers $5 \times 6 = 30$

Thirdly we will form single digit natural numbers, which are given in the question they are five in number, 5 because zero cannot be included because we need natural numbers.

Hence the total number of numbers formed which are less than 1000 and are formed by given numbers are

180+30+5 = 215.

40. Question

How many five digit telephone numbers can be constructed using the digits 0 to 9. If each number starts with 67 and no digit appears more than once?

Answer

We have to find the possible five digit numbers that are formed by the numbers 0 to 9 which starts from 67 which means first two digits are 6 and 7, when repetition of digits is not allowed.

We will use the concept of multiplication because there are sub jobs dependent on each other and are performed one after the other.

The first position from left is occupied by 6 and second position is occupied by 7 , now we have to make choices on the third , fourth and fifth position.

The third position has eight choices because out of ten 6, and 7 numbers have been used which means we are left with eight choices, for the fourth position we have seven choices because one is used in the third position, for the fifth position we have six choices left.

The number of ways in which we can form five digit numbers when two digits are fixed, and the repetition of digits is not allowed, along with given numbers $1 \times 1 \times 8 \times 7 \times 6 = 336$.

41. Question

Find the number of ways in which 8 distinct toys can be distributed among 5 children.

Answer

We have to find the possible number of ways in which we can distribute eight toys among five students when repetition of distribution of toys is allowed.

We will use the concept of multiplication because there are eight sub jobs dependent on each other and are performed one after the other.

The thing that is distributed is considered to have choices, not the things to which we have to give them; it means that in this problem the toys have choices more precisely five choices are there for each toy and children won’t choose any because toys have the right to choose.

The number of ways in which we can distribute toys among five children where repetition of distribution is allowed $5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 = 5^8 = 390625$

42. Question

Find the number of ways in which one can post 5 letters in 7 letter boxes.

Answer

We have to find the possible number of ways in which we can post five letters among seven letter boxes when repetition of distribution of letters is allowed.

We will use the concept of multiplication because there are five sub jobs dependent on each other and are performed one after the other.

The thing that is distributed is considered to have choices, not the things to which we have to give them; it means that in this problem the letters have choices more precisely seven choices are there for each letter and letter boxes won’t choose any because letters have the right to choose.

The number of ways in which we can post letters among seven letter boxes where repetition of distribution is allowed $7 \times 7 \times 7 \times 7 \times 7 = 7^5 = 16807$.

43. Question

Three dice are rolled. Find the number of possible outcomes in which at least one die shows 5.

Answer

We have to find the number of total outcomes when three dices are rolled the outcomes can be different and can be the same, but one outcome of at least one dice is made fixed.

We will use the concept of multiplication because there are three sub jobs dependent on each other and one job is performed one after the other.

The choices of outcomes from one dice are six, there are three dices so choices for each dice will be six but the outcome of at least one dice is made fixed which means at most all dices would have a fixed outcome which is 5.

The number of outcomes which are formed when three dices are rolled one after the other but at least one dice is fixed at 5

= total number of possible outcomes – the number of possible outcomes in which 5 does not occur on any dice.

$= 6 \times 6 \times 6 - 5 \times 5 \times 5 = 216 – 125 = 91$

A number of possible outcomes in which 5 does not occur on any dice means that we have deducted 5 from all the six possible outcomes and now only five possible outcomes are there, which will give 125 total outcomes.

44. Question

Find the total number of ways in which 20 balls can be put into 5 boxes, so that first box contains just one ball.

Answer

We have to find the possible number of ways in which we can put twenty balls in five boxes so that the first box contains only one ball when repetition of distribution of balls is allowed.

We will use the concept of multiplication because there are twenty sub jobs dependent on each other and are performed one after the other.

The thing that is distributed is considered to have choices not the things to which we have to give them, it means that in this problem the balls have choices more precisely four choices are there for each ball and boxes won't choose any because letters have the right to choose. But one box has the right to choose any one of the twenty balls, so the first box has twenty choices.

The number of ways in which we can put nineteen balls in four boxes where repetition of distribution is allowed

$4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 = 4^{19}$

Hence the total number of ways in which we can put twenty balls in five boxes such that first box contains only one ball $= 20 \times 4^{19}$.

ILLUSTRATION –

To illustrate suppose there are five balls a, b, c, d, e to be put in three boxes A, B, C such that first box contains only one ball.

Let the first box be A and suppose we have combination of a, A , so the rest combinations would be $2 \times 2 \times 2 \times 2 = 2^4$, but we have more combinations like b, A ; c, A ; d, A ; e, A these combinations are five , so we add them at the end, and the final answer we get is $= 5 \times 2^4$.

45. Question

In how many ways can 5 different balls be distributed among three boxes?

Answer

We have to find the possible number of ways in which we can put five balls in three boxes when repetition of distribution of balls is allowed.

We will use the concept of multiplication because there are five sub jobs dependent on each other and are performed one after the other.

The thing that is distributed is considered to have choices, not the things to which we have to give them; it means that in this problem the balls have choices more precisely three choices are there for each ball and boxes won't choose any because balls have the right to choose.

The number of ways in which we can put five balls among three boxes where repetition of distribution is allowed $3 \times 3 \times 3 \times 3 \times 3 = 3^5 = 243$.

46. Question

In how many ways can 7 letters be posted in 4 letter boxes?

Answer

We have to find the possible number of ways in which we can post seven letters among four letter boxes when repetition of distribution of letters is allowed.

We will use the concept of multiplication because there are seven sub jobs dependent on each other and are performed one after the other.

The thing that is distributed is considered to have choices, not the things to which we have to give them; it means that in this problem the letters have choices more precisely four choices are there for each letter and letter boxes won't choose any because letters have the right to choose.

The number of ways in which we can post letters among four letter boxes where repetition of distribution is allowed $4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 = 4^7$.

47. Question

In how many ways can 4 prizes be distributed among 5 students, when

i. no student gets more than one prize?

ii. a student may get any number of prizes?

iii. no student may gets all the prizes?

Answer

(i). We have to find the possible number of ways in which we can give four prizes among five students when no boy gets more than one price which means that there is no repetition.

We will use the concept of multiplication because there are four sub jobs dependent on each other and are performed one after the other.

The thing that is distributed is considered to have choices not the things to which we have to give them, it means that in this problem the prizes have choices more precisely first prize will have five choices, second prize will have four choices and the choices will keep on decreasing by one as we go on giving prizes, and students won't choose any because prizes will have the right to choose.

The number of ways in which we can give four prizes among five students where repetition of distribution is not allowed $5 \times 4 \times 3 \times 2 = 5! = 120$.

(ii). We have to find the possible number of ways in which we can give four prizes among five students when any student can get any number of prices which means that there is a repetition of prizes.

We will use the concept of multiplication because there are four sub jobs dependent on each other and are performed one after the other.

The thing that is distributed is considered to have choices, not the things to which we have to give them; it means that in this problem the prizes have choices more precisely five choices are there for each prize and students won't choose any because prizes have the right to choose.

The number of ways in which we can give four prizes among five students where repetition of distribution is allowed $5 \times 5 \times 5 \times 5 = 5^4 = 625$.

(iii). We have to find the possible number of ways in which we can give four prizes among five students when no student gets all the prizes.

We will use the concept of multiplication because there are four sub jobs dependent on each other and are performed one after the other.

The thing that is distributed is considered to have choices, not the things to which we have to give them; it means that in this problem the prizes have choices more precisely five choices are there for each prize and students won't choose any because prizes have the right to choose.

The number of ways in which we can give four prizes among five students when no student gets all the prices

= number of ways in which any student can get any number of prices – the number of ways in which one student gets all the prizes.

= 64 – 5 = 59.

There are five students, so any one student will get all the prizes hence there are five choices to give all the prizes to any one student.

48. Question

There are 10 lamps in a hall. Each one of them can be switched on independently. find the number of ways in which the hall can be illuminated.

Answer

We have to find the possible number of ways in which we can illuminate the hall by lighting the lamps which are independent of each other.

We will use the concept of multiplication because there are four sub jobs dependent on each other and are performed one after the other.

We will use a combination technique in this question, which will make us solve the question in an easy way. The combination means selection in rough language.

The combination is denoted by the symbol $^{\nu}C_r$, which means a number of ways of selecting r objects at a time from n objects.

We will make cases by choosing a specific number of lamps at one time,

Taking one lamp at a time, we can enlighten the hall in $^{10}C_1$ ways.

Taking two lamps at a time, we can enlighten the hall in $^{10}C_2$ ways.

Taking three lamps at a time, we can enlighten the hall in $^{10}C_3$ ways

Taking four lamps at a time, we can enlighten the hall in $^{10}C_4$ ways.

Taking five lamps at a time, we can enlighten the hall in $^{10}C_5$ ways.

Taking six lamps at a time, we can enlighten the hall in $^{10}C_6$ ways.

Taking seven lamps at a time, we can enlighten the hall in $^{10}C_7$ ways.

Taking eight lamps at a time, we can enlighten the hall in $^{10}C_8$ ways.

Taking nine lamps at a time, we can enlighten the hall in$^{10}C_9$ ways.

Taking ten lamps at a time, we can enlighten the hall in$^{10}C_{10}$ ways.

By using the binomial expansion $(1+x)^{10}$, if we put x = 1, then we will get the sum of these binomial coefficients in which there is extra $^{10}C_0$ term which is equal to 1, so we subtract 1 from the sum.

Therefore the total number of ways of enlightening the hall are = 2^{10} – 1

Exercise 16.3

1 A. Question

Evaluate each of the following:

$^8P_3$

Answer

To find: $^8P_3$

$^8P_3$ can be written as P(8,3)

We know,

$$P(n,r) = \frac{n!}{(n-r)!}$$

$$P(8,3) = \frac{8!}{(8-3)!}$$

$$= \frac{8!}{5!}$$

$$= \frac{8 \times 7 \times 6 \times 5!}{5!}$$

= 8×7×6

= 336

Hence, $^8P_3$ = 336

1 B. Question

Evaluate each of the following:

$^{10}P_4$

Answer

To find: $^{10}P_4$

$^{10}P_4$ can be written as P(10,4)

We know,

$$P(n,r) = \frac{n!}{(n-r)!}$$

$$P(10,4) = \frac{10!}{(10-4)!}$$

$$= \frac{10!}{6!}$$

$$= \frac{10 \times 9 \times 8 \times 7 \times 6!}{6!}$$

= 10×9×8×7

= 5040

Hence, $^{10}P_4$ = 5040

1 C. Question

Evaluate each of the following:

$^6P_6$

Answer

To find: $^6P_6$

$^6P_6$ can be written as P(6,6)

We know,

$$P(n,r) = \frac{n!}{(n-r)!}$$

$$P(6,6) = \frac{6!}{(6-6)!}$$

$$= \frac{6!}{0!}$$

{∵ 0! = 1}

$$= \frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{1}$$

= 720

Hence, $^6P_6$ = 720

1 D. Question

Evaluate each of the following:

P(6, 4)

Answer

To find: P(6, 4)

We know,

$$P(n,r) = \frac{n!}{(n-r)!}$$

$$P(6,4) = \frac{6!}{(6-4)!}$$

$$= \frac{6!}{2!}$$

$$= \frac{6 \times 5 \times 4 \times 3 \times 2!}{2!}$$

= 6×5×4×3

= 360

Hence, $^6P_4$ = 360

2. Question

If P (5, r) = P (6, r – 1), find r.

Answer

Given: P (5, r) = P (6, r – 1)

To find: value of r

We know,

$$P(n,r) = \frac{n!}{(n-r)!}$$

$$P(5,r) = \frac{5!}{(5-r)!}$$

$$P(6,r-1) = \frac{6!}{(6-(r-1))!}$$

$$= \frac{6!}{(6-r+1)!}$$

$= \frac{6!}{(7-r)!}$

So, according to question:

P (5, r) = P (6, r – 1)

$\Rightarrow \frac{5!}{(5-r)!} = \frac{6!}{(7-r)!}$

$\Rightarrow \frac{(7-r)!}{(5-r)!} = \frac{6!}{5!}$

$\Rightarrow \frac{(7-r)(7-r-1)(7-r-2)!}{(5-r)!} = \frac{6 \times 5!}{5!}$

$\Rightarrow \frac{(7-r)(6-r)(5-r)!}{(5-r)!} = 6$

⇒ (7 – r) (6 – r) = 6

⇒ 42 – 6r – 7r + r^2 = 6

⇒ 42 – 6 – 13r + r^2 = 0

⇒ r^2 – 13r + 36 = 0

⇒ r^2 – 9r – 4r + 36 = 0

⇒ r(r – 9) – 4(r – 9) = 0

⇒ (r – 9) (r – 4) = 0

⇒ r = 9 or 4

For, P(n, r): r ≤ n

∴ **r = 4** {for, P(5, r)}

3. Question

If 5 P(4, n) = 6 P(5, n – 1), find n.

Answer

Given: 5 P(4, n) = 6 P(5, n – 1)

To find: value of n

We know,

$P(n,r) = \frac{n!}{(n-r)!}$

$P(4,n) = \frac{4!}{(4-n)!}$

$P(5,n-1) = \frac{5!}{(5-(n-1))!}$

$= \frac{5!}{(5-n+1)!}$

$= \frac{5!}{(6-n)!}$

So, according to question:

5 P(4, n) = 6 P(5, n – 1)

$\Rightarrow 5 \times \frac{4!}{(4-n)!} = 6 \times \frac{5!}{(6-n)!}$

$\Rightarrow \frac{(6-n)!}{(4-n)!} = \frac{6}{5} \times \frac{5!}{4!}$

$\Rightarrow \frac{(6-n)(6-n-1)(6-n-2)!}{(4-n)!} = \frac{6 \times 5 \times 4!}{5 \times 4!}$

$\Rightarrow \frac{(6-n)(5-n)(4-n)!}{(4-n)!} = 6$

⇒ (6 – n) (5 – n) = 6

⇒ 30 – 6n – 5n + n^2 = 6

⇒ 30 – 6 – 11n + n^2 = 0

⇒ n^2 – 11n + 24 = 0

⇒ n^2 – 8n – 3n + 24 = 0

⇒ n(n – 8) – 3(n – 8) = 0

⇒ (n – 8) (n – 3) = 0

⇒ n = 8 or 3

For, P(n, r): r ≤ n

∴ **n = 3** {for, P(4, n)}

4. Question

If P(n, 5) = 20 P(n, 3), find n.

Answer

Given: P(n, 5) = 20 P(n, 3)

To find: value of n

We know,

$P(n,r) = \frac{n!}{(n-r)!}$

$P(n,5) = \frac{n!}{(n-5)!}$

$P(n,3) = \frac{n!}{(n-3)!}$

So, according to question:

P(n, 5) = 20 P(n, 3)

$\Rightarrow \frac{n!}{(n-5)!} = 20 \times \frac{n!}{(n-3)!}$

$\Rightarrow \frac{n!\,(n-3)!}{n!\,(n-5)!} = 20$

$\Rightarrow \frac{(n-3)(n-3-1)(n-3-2)!}{(n-5)!} = 20$

$\Rightarrow \frac{(n-3)(n-4)(n-5)!}{(n-5)!} = 20$

$\Rightarrow (n - 3)(n - 4) = 20$

$\Rightarrow n^2 - 3n - 4n + 12 = 20$

$\Rightarrow n^2 - 7n + 12 - 20 = 0$

$\Rightarrow n^2 - 7n - 8 = 0$

$\Rightarrow n^2 - 8n + n - 8 = 0$

$\Rightarrow n(n - 8) - 1(n - 8) = 0$

$\Rightarrow (n - 8)(n - 1) = 0$

$\Rightarrow n = 8$ or 1

For, P(n, r): $n \geq r$

$\therefore$ **n = 8** {for, P(n, 5)}

5. Question

If $^nP_4 = 360$, find the value of n.

Answer

Given: $^nP_4 = 360$

To find: value of n

$^nP_4$ can be written as P(n, 4)

We know,

$$P(n, r) = \frac{n!}{(n - r)!}$$

$$P(n, 4) = \frac{n!}{(n - 4)!}$$

So, according to question:

$^nP_4 = P(n, 4) = 360$

$$\Rightarrow \frac{n!}{(n - 4)!} = 360$$

$$\Rightarrow \frac{n(n - 1)(n - 2)(n - 3)(n - 4)!}{(n - 4)!} = 360$$

$\Rightarrow n(n - 1)(n - 2)(n - 3) = 360$

$\Rightarrow n(n - 1)(n - 2)(n - 3) = 6 \times 5 \times 4 \times 3$

On comparing:

The value of n = 6

6. Question

If P(9, r) = 3024, find r.

Answer

Given: P(9, r) = 3024

To find: value of r

We know,

$$P(n,r) = \frac{n!}{(n-r)!}$$

$$P(9,r) = \frac{9!}{(9-r)!}$$

So, according to question:

P(9, r) = 3024

$$\Rightarrow \frac{9!}{(9-r)!} = 3024$$

$$\Rightarrow \frac{1}{(9-r)!} = \frac{3024}{9!}$$

$$\Rightarrow \frac{1}{(9-r)!} = \frac{3024}{9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}$$

$$\Rightarrow \frac{1}{(9-r)!} = \frac{3024}{3024 \times 5 \times 4 \times 3 \times 2 \times 1}$$

$$\Rightarrow \frac{1}{(9-r)!} = \frac{1}{5!}$$

⇒ (9 – r)! = 5!

⇒ 9 – r = 5

⇒ -r = 5 – 9

⇒ -r = -4

∴ The value of r = 4

7. Question

If P(11, r) = P (12, r – 1), find n.

Answer

Given: P(11, r) = P (12, r – 1)

To find: value of n

We know,

$$P(n,r) = \frac{n!}{(n-r)!}$$

$$P(11,r) = \frac{11!}{(11-r)!}$$

$$P(12,r-1) = \frac{12!}{(12-(r-1))!}$$

$$= \frac{12!}{(12-r+1)!}$$

$$= \frac{12!}{(13-r)!}$$

So, according to question:

P (11, r) = P (12, r – 1)

$$\Rightarrow \frac{11!}{(11-r)!} = \frac{12!}{(13-r)!}$$

$$\Rightarrow \frac{(13-r)!}{(11-r)!} = \frac{12!}{11!}$$

$$\Rightarrow \frac{(13-r)(13-r-1)(13-r-2)!}{(11-r)!} = \frac{12 \times 11!}{11!}$$

$$\Rightarrow \frac{(13-r)(12-r)(11-r)!}{(11-r)!} = 12$$

⇒ (13 – r) (12 – r) = 12

⇒ 156 – 12r – 13r + r^2 = 12

⇒ 156 – 12 – 25r + r^2 = 0

⇒ r^2 – 25r + 144 = 0

⇒ r^2 – 16r – 9r + 144 = 0

⇒ r(r – 16) – 9(r – 16) = 0

⇒ (r – 9) (r – 16) = 0

⇒ r = 9 or 16

For, P(n, r): r ≤ n

∴ **r = 9** {for, P(11, r)}

8. Question

If P(n, 4) = 12. P(n, 2), find n.

Answer

Given: P(n, 4) = 12. P(n, 2)

To find: value of n

We know,

$$P(n,r) = \frac{n!}{(n-r)!}$$

$$P(n,4) = \frac{n!}{(n-4)!}$$

$$P(n,2) = \frac{n!}{(n-2)!}$$

So, according to question:

P(n, 4) = 12 P(n, 2)

$$\Rightarrow \frac{n!}{(n-4)!} = 12 \times \frac{n!}{(n-2)!}$$

$$\Rightarrow \frac{n!\,(n-2)!}{n!\,(n-4)!} = 12$$

$$\Rightarrow \frac{(n-2)(n-2-1)(n-2-2)!}{(n-4)!} = 12$$

$$\Rightarrow \frac{(n-2)(n-3)(n-4)!}{(n-4)!} = 12$$

⇒ (n – 2) (n – 3) = 12

⇒ n^2 – 3n – 2n + 6 = 12

$\Rightarrow n^2 - 5n + 6 - 12 = 0$

$\Rightarrow n^2 - 5n - 6 = 0$

$\Rightarrow n^2 - 6n + n - 6 = 0$

$\Rightarrow n(n - 6) - 1(n - 6) = 0$

$\Rightarrow (n - 6)(n - 1) = 0$

$\Rightarrow n = 6$ or 1

For, P(n, r): $n \geq r$

∴ **n = 6** {for, P(n, 4)}

9. Question

If P(n – 1, 3) : P(n, 4) = 1 : 9, find n.

Answer

Given: P(n – 1, 3) : P(n, 4) = 1 : 9

To find: value of n

We know,

$$P(n,r) = \frac{n!}{(n-r)!}$$

$$P(n-1,3) = \frac{(n-1)!}{(n-1-3)!} = \frac{(n-1)!}{(n-4)!}$$

$$P(n,4) = \frac{n!}{(n-4)!}$$

So, according to question:

P(n – 1, 3) : P(n, 4) = 1 : 9

$$\Rightarrow \frac{P(n-1,3)}{P(n,4)} = \frac{1}{9}$$

$$\Rightarrow \frac{\frac{(n-1)!}{(n-4)!}}{\frac{n!}{(n-4)!}} = \frac{1}{9}$$

$$\Rightarrow \frac{(n-1)!}{(n-4)!} \times \frac{(n-4)!}{n!} = \frac{1}{9}$$

$$\Rightarrow \frac{(n-1)!}{n!} = \frac{1}{9}$$

$$\Rightarrow \frac{(n-1)!}{n(n-1)!} = \frac{1}{9}$$

$$\Rightarrow \frac{1}{n} = \frac{1}{9}$$

$\Rightarrow n = 9$

∴ The value of n = 9

10. Question

If P(2n – 1, n) : P(2n + 1, n – 1) = 22 : 7 find n.

Answer

Given: P(2n – 1, n) : P(2n + 1, n – 1) = 22 : 7

To find: value of n

We know,

$$P(n,r) = \frac{n!}{(n-r)!}$$

$$P(2n-1,n) = \frac{(2n-1)!}{(2n-1-n)!} = \frac{(2n-1)!}{(n-1)!}$$

$$P(2n+1,n-1) = \frac{(2n+1)!}{(2n+1-n+1)!} = \frac{(2n+1)!}{(n+2)!}$$

So, according to question:

P(2n – 1, n) : P(2n + 1, n – 1) = 22 : 7

$$\Rightarrow \frac{P(2n-1,n)}{P(2n+1,n-1)} = \frac{22}{7}$$

$$\Rightarrow \frac{\frac{(2n-1)!}{(n-1)!}}{\frac{(2n+1)!}{(n+2)!}} = \frac{22}{7}$$

$$\Rightarrow \frac{(2n-1)!}{(n-1)!} \times \frac{(n+2)!}{(2n+1)!} = \frac{22}{7}$$

$$\Rightarrow \frac{(2n-1)!}{(n-1)!} \times \frac{(n+2)(n+2-1)(n+2-2)(n+2-3)!}{(2n+1)(2n+1-1)(2n+1-2)!} = \frac{22}{7}$$

$$\Rightarrow \frac{(2n-1)!}{(n-1)!} \times \frac{(n+2)(n+1)n(n-1)!}{(2n+1)2n(2n-1)!} = \frac{22}{7}$$

$$\Rightarrow \frac{(n+2)(n+1)}{(2n+1)2} = \frac{22}{7}$$

$\Rightarrow 7(n + 2)(n + 1) = 22\times2\ (2n + 1)$

$\Rightarrow 7(n^2 + n + 2n + 2) = 88n + 44$

$\Rightarrow 7(n^2 + 3n + 2) = 88n + 44$

$\Rightarrow 7n^2 + 21n + 14 = 88n + 44$

$\Rightarrow 7n^2 + 21n - 88n + 14 - 44 = 0$

$\Rightarrow 7n^2 - 67n - 30 = 0$

$\Rightarrow 7n^2 - 70n + 3n - 30 = 0$

$\Rightarrow 7n(n - 10) + 3(n - 10) = 0$

$\Rightarrow (n - 10)(7n + 3) = 0$

$$\Rightarrow n = 10, \frac{-3}{7}$$

$$\text{As, } n \neq \frac{-3}{7}$$

∴ The value of n = 10

11. Question

If P(n, 5) : P(n, 3) = 2 : 1, find n.

Answer

Given: P(n, 5) : P(n, 3) = 2 : 1

To find: value of n

We know,

$$P(n,r) = \frac{n!}{(n-r)!}$$

$$P(n,5) = \frac{n!}{(n-5)!}$$

$$P(n,3) = \frac{n!}{(n-3)!}$$

So, according to question:

P(n, 5) : P(n, 3) = 2 : 1

$$\Rightarrow \frac{P(n,5)}{P(n,3)} = \frac{2}{1}$$

$$\Rightarrow \frac{\frac{n!}{(n-5)!}}{\frac{n!}{(n-3)!}} = \frac{2}{1}$$

$$\Rightarrow \frac{n!}{(n-5)!} \times \frac{(n-3)!}{n!} = \frac{2}{1}$$

$$\Rightarrow \frac{(n-3)!}{(n-5)!} = \frac{2}{1}$$

$$\Rightarrow \frac{(n-3)(n-3-1)(n-3-2)!}{(n-5)!} = \frac{2}{1}$$

$$\Rightarrow \frac{(n-3)(n-4)(n-5)!}{(n-5)!} = \frac{2}{1}$$

⇒ (n – 3)(n – 4) = 2

⇒ n^2 – 3n – 4n + 12 = 2

⇒ n^2 – 7n + 12 – 2 = 0

⇒ n^2 – 7n + 10 = 0

⇒ n^2 – 5n – 2n + 10 = 0

⇒ n(n – 5) – 2(n – 5) = 0

⇒ (n – 5) (n – 2) = 0

⇒ n = 5 or 2

For, P(n, r): n ≥ r

∴ **n = 5** {∵ P(n, 5)}

12. Question

Prove that:

1. P(1, 1) + 2. P(2, 2) + 3 . P(3, 3) + ... + n . P(n, n) = P(n + 1, n + 1) – 1.

Answer

To prove: P(1, 1) + 2. P(2, 2) + 3 . P(3, 3) + ... + n . P(n, n) = P(n + 1, n + 1) – 1

We know,

$$P(n,r) = \frac{n!}{(n-r)!}$$

$$\Rightarrow P(n,n) = \frac{n!}{(n-n)!}$$

$$\Rightarrow P(n,n) = \frac{n!}{(0)!} = n!$$

Take L.H.S.:

1. P(1, 1) + 2. P(2, 2) + 3. P(3, 3) + ... + n . P(n, n)

= 1.1! + 2.2! + 3.3! +.........+ n.n!

{∵ P(n, n) = n!}

$$= \sum_{r=1}^{n} r.r!$$

$$= \sum_{r=1}^{n} r.r! + r! - r!$$

$$= \sum_{r=1}^{n} (r+1)r! - r!$$

$$= \sum_{r=1}^{n} (r+1)! - r!$$

= (2! – 1!) + (3! – 2!) + (4! – 3!) + ... + (n! – (n – 1)!) + ((n+1)! – n!)

= 2! – 1! + 3! – 2! + 4! – 3! + ... + n! – (n – 1)! + (n+1)! – n!

= (n + 1)! – 1!

= (n + 1)! – 1

{∵ P(n, n) = n!}

= P(n+1, n+1) – 1

= R.H.S

Hence Proved

13. Question

If P(15, r – 1) : P(16, r – 2) = 3 : 4, find r.

Answer

Given: P(15, r – 1) : P(16, r – 2) = 3 : 4

To find: value of r

We know,

$$P(n,r) = \frac{n!}{(n-r)!}$$

$$P(15,r-1) = \frac{15!}{(15-r+1)!} = \frac{15!}{(16-r)!}$$

$$P(16, r-2) = \frac{16!}{(16-r+2)!} = \frac{16!}{(18-r)!}$$

So, according to question:

P(15, r – 1) : P(16, r – 2) = 3 : 4

$$\Rightarrow \frac{P(15, r-1)}{P(16, r-2)} = \frac{3}{4}$$

$$\Rightarrow \frac{\frac{15!}{(16-r)!}}{\frac{16!}{(18-r)!}} = \frac{3}{4}$$

$$\Rightarrow \frac{15!}{(16-r)!} \times \frac{(18-r)!}{16!} = \frac{3}{4}$$

$$\Rightarrow \frac{15!}{(16-r)!} \times \frac{(18-r)(18-r-1)(18-r-2)!}{16 \times 15!} = \frac{3}{4}$$

$$\Rightarrow \frac{1}{(16-r)!} \times \frac{(18-r)(17-r)(16-r)!}{16} = \frac{3}{4}$$

$$\Rightarrow (18-r)(17-r) = \frac{3}{4} \times 16$$

⇒ (18 – r) (17 – r) = 12

⇒ 306 – 18r – 17r + r^2 = 12

⇒ 306 – 12 – 35r + r^2 = 0

⇒ r^2 – 35r + 294 = 0

⇒ r^2 – 21r – 14r + 294 = 0

⇒ r(r – 21) – 14(r – 21) = 0

⇒ (r – 14) (r – 21) = 0

⇒ r = 14 or 21

For, P(n, r): r ≤ n

∴ **r = 14** {for, P(15, r – 1)}

14. Question

If $^{n+5}P_{n+1} = \frac{11(n-1)}{2}\, ^{n+3}P_n$, find n.

Answer

Given: $P(n+5, n+1) = \frac{11(n-1)}{2} P(n+3, n)$

To find: value of n

We know,

$$P(n, r) = \frac{n!}{(n-r)!}$$

$$P(n+5, n+1) = \frac{(n+5)!}{(n+5-n-1)!} = \frac{(n+5)!}{4!}$$

$$P(n+3,n) = \frac{(n+3)!}{(n+3-n)!} = \frac{(n+3)!}{3!}$$

So, according to question:

$$P(n+5,n+1) = \frac{11(n-1)}{2} P(n+3,n)$$

$$\Rightarrow \frac{(n+5)!}{4!} = \frac{11(n-1)}{2} \times \frac{(n+3)!}{3!}$$

$$\Rightarrow \frac{(n+5)!}{(n-1)(n+3)!} = \frac{11}{2} \times \frac{4!}{3!}$$

$$\Rightarrow \frac{(n+5)(n+5-1)(n+5-2)!}{(n-1)(n+3)!} = \frac{11}{2} \times \frac{4 \times 3!}{3!}$$

$$\Rightarrow \frac{(n+5)(n+4)(n+3)!}{(n-1)(n+3)!} = \frac{44}{2}$$

$$\Rightarrow \frac{(n+5)(n+4)}{(n-1)} = 22$$

⇒ (n + 5) (n + 4) = 22 (n – 1)

⇒ n^2 + 4n + 5n + 20 = 22n – 22

⇒ n^2 + 9n + 20 - 22n + 22 = 0

⇒ n^2 – 13n + 42 = 0

⇒ n^2 – 6n – 7n + 42 = 0

⇒ n(n – 6) – 7(n – 6) = 0

⇒ (n – 7) (n – 6) = 0

⇒ n = 7 or 6

∴ The value of n can either be 6 or 7

15. Question

In how many ways can five children stand in a queue?

Answer

Given: There are five children

To find: The number of ways in which these children can stand in a queue

Formula used:

Number of arrangements of n things taken all at a time = P(n, n)

$$P(n,r) = \frac{n!}{(n-r)!}$$

∴The total number of ways in which five children can stand in a queue

= the number of arrangements of 5 things taken all at a time

= P(5, 5)

$$= \frac{5!}{(5-5)!}$$

$$= \frac{5!}{0!}$$

$\{\because 0! = 1\}$

$= 5!$

$= 5 \times 4 \times 3 \times 2 \times 1$

$= 120$

Hence, Number of ways in which five children can stand in a queue are 120

16. Question

From among the 36 teachers in a school, one principal and one vice-principal are to be appointed. In how many ways can this be done?

Answer

Given, the total number of teachers in a school = 36

To find: The number of ways in which one principal and one vice-principal can be appointed

Formula used:

Number of arrangements of n things taken r at a time = P(n, r)

$$P(n,r) = \frac{n!}{(n-r)!}$$

∴ The total number of ways in which this can be done

= the number of arrangements of 36 things taken 2 at a time

= P(36, 2)

$$= \frac{36!}{(36-2)!}$$

$$= \frac{36!}{34!}$$

$$= \frac{36 \times 35 \times 34!}{34!}$$

$= 36 \times 35$

$= 1260$

Hence, Number of ways in which one principal and one vice-principal are to be appointed out of total 36 teachers in school are 1260

17. Question

Four letters E, K, S and V, one in each, were purchased from a plastic warehouse. How many ordered pairs of letters, to be used as initials, can be formed from them?

Answer

Given: the total number of letters = 4

To find: Number of ordered pairs of letters that can be formed like (E, K) or (S, E) etc.

Formula used:

Number of arrangements of n things taken r at a time = P(n, r)

$$P(n,r) = \frac{n!}{(n-r)!}$$

∴ The total number of ways in which this can be done

= the number of arrangements of 4 things taken 2 at a time

= P(4, 2)

$= \frac{4!}{(4-2)!}$

$= \frac{4!}{2!}$

$= \frac{4 \times 3 \times 2!}{2!}$

= 4 × 3

= 12

Hence, Number of ways in which ordered pairs of letters, to be used as initials, can be formed from given 4 letters are 12

18. Question

Four books, one each in Chemistry, Physics, Biology and Mathematics, are to be arranged in a shelf. In how many ways can this be done?

Answer

Given: the total number of books = 4

To find: Number of ways in which these 4 books can be arranged in a shelf

Formula used:

Number of arrangements of n things taken all at a time = P(n, n)

$P(n,r) = \frac{n!}{(n-r)!}$

∴ The total number of ways

= the number of arrangements of 4 things taken all at a time

= P(4, 4)

$= \frac{4!}{(4-4)!}$

$= \frac{4!}{0!}$

{∵ 0! = 1}

= 4 × 3 × 2 × 1

= 24

Hence, Number of ways in which these 4 books can be arranged in a shelf are 24

19. Question

Find the number of different 4-letter words, with or without meanings that can be formed from the letters of the word ‘NUMBER’.

Answer

Given, the total number of letters in ‘NUMBER’ = 6

To find: Number of different 4-letter words, with or without meanings like numb, nume or nmbr etc.

Formula used:

Number of arrangements of n things taken r at a time = P(n, r)

$P(n,r) = \frac{n!}{(n-r)!}$

∴ The total number of ways in which this can be done

= the number of arrangements of 6 things taken 4 at a time

= P(6, 4)

$= \frac{6!}{(6-4)!}$

$= \frac{6!}{2!}$

$= \frac{6 \times 5 \times 4 \times 3 \times 2!}{2!}$

= 6 × 5 × 4 × 3

= 360

Hence, Number of different 4-letter words, with or without meanings that can be formed from the letters of the word 'NUMBER' are 360

20. Question

How many three-digit numbers are there, with distinct digits, with each digit odd?

Answer

Given: The odd digits are: 1, 3, 5, 7 and 9.

∴ The total number of odd-digits = 5

To find: Total number of three-digit numbers, with distinct digits, with each digit odd like 135 or 159 etc.

Formula used:

Number of arrangements of n things taken r at a time = P(n, r)

$P(n,r) = \frac{n!}{(n-r)!}$

∴The total number of ways in which this can be done

= the number of arrangements of 5 things taken 3 at a time

= P(5, 3)

$= \frac{5!}{(5-3)!}$

$= \frac{5!}{2!}$

$= \frac{5 \times 4 \times 3 \times 2!}{2!}$

= 5 × 4 × 3

= 60

Hence, total number of three-digit numbers, with distinct digits, with each digit odd are 60

21. Question

How many words, with or without meaning, can be formed by using all the letters of the word 'DELHI', using each letter exactly once?

Answer

Given: the total number of letters in 'DELHI' = 5

To find: Number of words, with or without meanings using all the letters of the word like eldhi or dehil etc.

Formula used:

Number of arrangements of n things taken all at a time = P(n, n)

$$P(n,r) = \frac{n!}{(n-r)!}$$

∴ The total number of ways in which this can be done

= the number of arrangements of 5 things taken all at a time

= P(5, 5)

$$= \frac{5!}{(5-5)!}$$

$$= \frac{5!}{0!}$$

{∵ 0! = 1}

= 5 × 4 × 3 × 2 × 1

= 120

Hence, number of words, with or without meanings using all the letters of the word 'DELHI' are 120

22. Question

How many words, with or without meaning, can be formed by using the letters of the word 'TRIANGLE'?

Answer

Given, the total number of letters in 'TRIANGLE' = 8

To find: number of words, with or without meanings using all the letters of the word like triangel or angletri etc.

Formula used:

Number of arrangements of n things taken all at a time = P(n, n)

$$P(n,r) = \frac{n!}{(n-r)!}$$

∴ The total number of ways in which this can be done

= the number of arrangements of 8 things taken all at a time

= P(8, 8)

$$= \frac{8!}{(8-8)!}$$

$$= \frac{8!}{0!}$$

{∵ 0! = 1}

= 8!

Hence, number of words, with or without meanings using all the letters of the word 'TRIANGLE' are 8!

23. Question

There are two works each of 3 volumes and two works each of 2 volumes; In how many ways can the 10 books be placed on a shelf so that the volumes of the same work are not separated?

Answer

Given: There are two works each of 3 volumes and two works each of 2 volumes

To find: Number of ways in which these books can be arranged in a shelf provided volumes of the same work are not separated

Let w_1, w_2, w_3, w_4, are four works

w_1 has n_1, n_2, n_3 as volumes

w_2 has m_1, m_2, m_3 as volumes

w_3 has a_1, a_2 as volumes

w_4 has b_1, b_2 as volumes

Now, firstly we have to arrange these 4 works like w_2 w_3 w_1 w_4 or w_1 w_2 w_4 w_3

This can be done in 4! ways

Now, we have to separately arrange volumes of these 4 works

w_1 has 3 volumes which can be arranged like n_2 n_1 n_3 or n_3 n_1 n_2

Volumes of w_1 can be arranged in 3! ways

Similarly,

Volumes of w_2 can be arranged in 3! ways

Volumes of w_3 can be arranged in 2! ways

Volumes of w_4 can be arranged in 2! Ways

$\therefore$ Total number of ways = $4! \times 3! \times 3! \times 2! \times 2!$

$= 24 \times 6 \times 6 \times 2 \times 2$

$= 3456$

Hence, the total number of ways in which these 10 books be placed on a shelf so that the volumes of the same work are not separated are 3456

24. Question

. There are 6 items in column A and 6 items in column B. A student is asked to match each item in column A with an item in column B. How many possible, correct or incorrect, answers are there to this question?

Answer

Given: The number of items in column A = 6 and in column B = 6

A student is asked to match each item in column A with an item in column B

To find: Possible number of correct or incorrect answers which he can give

Let the items of column A are fixed i.e. they arrangement is not changing

Column A	Column B
A_1	
A_2	
A_3	
A_4	
A_5	
A_6	

Now, we just have to arrange items of column B

Formula used:

Number of arrangements of n things taken all at a time = P(n, n)

$$P(n,r) = \frac{n!}{(n-r)!}$$

∴ The total number of ways in which this can be done

= the number of arrangements of 6 things taken all at a time

= P(6, 6)

$$= \frac{6!}{(6-6)!}$$

$$= \frac{6!}{0!}$$

{∵ 0! = 1}

= 6 × 5 × 4 × 3 × 2 × 1

= 720

Hence, possible number of correct or incorrect answers which a student can give are 720

25. Question

How many three-digit numbers are there, with no digit repeated?

Answer

Given, digits which can be used to make numbers are 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9

The number of these digits are 10

To find: Total number of three-digit numbers with no digit repeated

Formula used:

Number of arrangements of n things taken r at a time = P(n, r)

$$P(n,r) = \frac{n!}{(n-r)!}$$

∴ The total number of ways

= the number of arrangements of 10 things taken 3 at a time

= P(10, 3)

$$= \frac{10!}{(10-3)!}$$

$$= \frac{10!}{7!}$$

$$= \frac{10 \times 9 \times 8 \times 7!}{7!}$$

= 10 × 9 × 8

= 720

But in these 720 numbers we also included those numbers which are starting from 0 like 023 or 056 etc. Being starting from 0, these are actually two-digit numbers. So, we need to subtract these numbers.

To find these numbers, fix the position of 0 at hundred’s place.

0		

Remaining numbers = 9 (1, 2, 3, 4, 5, 6, 7, 8 or 9)

Arrange these 9 numbers in remaining 2 places.

Formula used:

Number of arrangements of n things taken r at a time = P(n, r)

$$P(n,r) = \frac{n!}{(n-r)!}$$

∴ The total numbers which are starting from 0 are =

= the number of arrangements of 9 things taken 2 at a time

= P(9, 2)

$$= \frac{9!}{(9-2)!}$$

$$= \frac{9!}{7!}$$

$$= \frac{9 \times 8 \times 7!}{7!}$$

= 9 × 8

= 72

Hence, total number of three-digit numbers with no digit repeated are, 720 – 72 = 648

26. Question

How many 6-digit telephone numbers can be constructed with digit 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 if each number starts with 35 and no digit appears more than once?

Answer

Given, numbers are 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9

The total number of these digits are 10

To find: 6-digit telephone numbers starting with 35 and no digit appears more than once

So, fix the position of first two digits as 3 and 5

3	5				

Now, we need to fill these 4 remaining places

Remaining number of digits are 8 (0, 1, 2, 4, 6, 7, 8, 9)

Arrange these 8 numbers at 4 places.

Formula used:

Number of arrangements of n things taken r at a time = P(n, r)

$$P(n,r) = \frac{n!}{(n-r)!}$$

∴ The total number of ways

= the number of arrangements of 8 things taken 4 at a time

= P(8, 4)

$$= \frac{8!}{(8-4)!}$$

$= \frac{8!}{4!}$

$= \frac{8 \times 7 \times 6 \times 5 \times 4!}{4!}$

= 8 × 7 × 6 × 5

= 1680

Hence, possible number of 6-digit telephone numbers starting with 35 are 1680

27. Question

In how many ways can 6 boys and 5 girls be arranged for a group photograph if the girls are to sit on chairs in a row and the boys are to stand in a row behind them?

Answer

Given: Number of boys = 6 and number of girls = 5

To find: Possible number of arrangements in a group photograph

Let boys be b_1, b_2, b_3, b_4, b_5, b_6 and girls be g_1, g_2, g_3, g_4, g_5

Possible arrangements are

b_1 b_2 b_3 b_5 b_6 b_4

g_2 g_4 g_1 g_5 g_3

b_2 b_1 b_5 b_3 b_4 b_6

g_2 g_4 g_5 g_1 g_3

In this arrangement, we are arranging boys and girls separately

Formula used:

Number of arrangements of n things taken all at a point = P(n, n)

$P(n, r) = \frac{n!}{(n-r)!}$

∴ Number of ways to arrange boys

= the number of arrangements of 6 things taken all at a time

= P(6, 6)

$= \frac{6!}{(6-6)!}$

$= \frac{6!}{0!}$

{∵ 0! = 1}

= 6!

= 6 × 5 × 4 × 3 × 2 × 1

= 720

Formula used:

Number of arrangements of n things taken all at a point = P(n, n)

$P(n, r) = \frac{n!}{(n-r)!}$

∴ Number of ways to arrange girls

= the number of arrangements of 5 things taken all at a time

= P(5, 5)

$= \frac{5!}{(5-5)!}$

$= \frac{5!}{0!}$

{∵ 0! = 1}

= 5!

= 5 × 4 × 3 × 2 × 1

= 120

Now, we will get total number of ways by multiplying their separate arrangements

∴ Total number of ways

= 720 × 120

= 86400

Hence, possible number of arrangements in which 6 boys and 5 girls can be arranged for a group photograph with provided conditions are 86400

28. Question

If a denotes the number of permutations of (x + 2) things taken all at a time, b the number of permutations of x things taken 11 at a time and c the number of permutations of x – 11 things taken all at a time such that a = 182 bc, find the value of x.

Answer

Given: a = 182 bc

To find: value of x

a denotes the number of permutations of (x + 2) things taken all at a time

∴ a = P(x+2, x+2)

{Number of arrangements of n things taken all at a time = P(n, n)}

b denotes the number of permutations of x things taken 11 at a time

∴ b = P(x, 11)

{Number of arrangements of n things taken r at a time = P(n,r)}

c denotes the number of permutations of x – 11 things taken all at a time

∴ c = P(x-11, x-11)

{Number of arrangements of n things taken all at a time = P(n,n)}

According to question:

a = 182 bc

⇒ P(x+2, x+2) = 182 × P(x, 11) × P(x-11, x-11)

$\left\{P(n,r) = \frac{n!}{(n-r)!}\right\}$

$\Rightarrow \frac{(x+2)!}{(x+2-x-2)!} = 182 \times \frac{x!}{(x-11)!} \times \frac{(x-11)!}{(x-11-x+11)!}$

$\Rightarrow \frac{(x+2)!}{0!} = 182 \times \frac{x!}{(x-11)!} \times \frac{(x-11)!}{0!}$

{∵ 0! = 1}

$\Rightarrow \frac{(x+2)(x+2-1)(x+2-2)!}{x!} = 182$

$\Rightarrow \frac{(x+2)(x+1)x!}{x!} = 182$

⇒ (x + 2) (x + 1) = 182

⇒ x^2 + 2x + x + 2 = 182

⇒ x^2 + 3x + 2 – 182 = 0

⇒ x^2 + 3x – 180 = 0

⇒ x^2 – 12x + 15x – 180 = 0

⇒ x(x – 12) + 15(x – 12) = 0

⇒ (x – 12) (x + 15) = 0

⇒ x = -15 or 12

⇒ x = 12 {x cannot hold negative value}

Hence, the value of x is 12

29. Question

How many 3-digit number can be formed by using the digits 1 to 9 if no digit is repeated?

Answer

Given: Digits which can be used to make numbers are 1, 2, 3, 4, 5, 6, 7, 8 and 9

The number of these digits are 9

To find: total number of three-digit numbers with no digit repeated

Formula used:

Number of arrangements of n things taken r at a time = P(n, r)

$P(n,r) = \frac{n!}{(n-r)!}$

∴ The total number of ways

= the number of arrangements of 9 things taken 3 at a time

= P(9, 3)

$= \frac{9!}{(9-3)!}$

$= \frac{9!}{6!}$

$= \frac{9 \times 8 \times 7 \times 6!}{6!}$

= 9 × 8 × 7

= 504

Hence, total number of three-digit numbers using digits 1 to 9 with no digit repeated are 504

30. Question

How many 3-digit even numbers can be made using the digits 1, 2, 3, 4, 5, 6, 7 if no digits is repeated?

Answer

Given: Digits which can be used to make numbers are 1, 2, 3, 4, 5, 6 and 7

The number of these digits are 7

To find: total number of three-digit even numbers with no digit repeated

Even numbers are those numbers whose unit's place is even

∴ fix the position of 1 even number at unit's place at one time

Even numbers are: 2, 4 and 6

Case 1:

Fix position of 2 at unit's place

		2

Remaining numbers = 6

Arrange these 6 numbers at remaining 2 places

Formula used:

Number of arrangements of n things taken r at a time = P(n, r)

$$P(n,r) = \frac{n!}{(n-r)!}$$

∴ The total numbers ending with 2 are =

= the number of arrangements of 6 things taken 2 at a time

= P(6, 2)

$$= \frac{6!}{(6-2)!}$$

$$= \frac{6!}{4!}$$

$$= \frac{6 \times 5 \times 4!}{4!}$$

= 6 × 5

= 30

Case 2:

Fix position of 4 at unit's place

		4

Remaining numbers = 6

Now, arrange these 6 numbers in remaining 2 places

Formula used:

Number of arrangements of n things taken r at a time = P(n, r)

$$P(n,r) = \frac{n!}{(n-r)!}$$

∴ The total numbers ending with 2 are =

= the number of arrangements of 6 things taken 2 at a time

= P(6, 2)

$= \frac{6!}{(6-2)!}$

$= \frac{6!}{4!}$

$= \frac{6 \times 5 \times 4!}{4!}$

= 6 × 5

= 30

Similarly,

When you fix position of 6 at unit's place, 30 more numbers will be formed.

Hence, total number of three-digit even numbers with no digit repeated are, 30 + 30 + 30 = 90

31. Question

Find the numbers of 4-digit numbers that can be formed using the digits 1, 2, 3, 4, 5, if no digit is repeated? How many of these will be even?

Answer

Given, digits which can be used to make numbers are 1, 2, 3, 4 and 5

The number of these digits are 5

To find: total number of 4-digit numbers with no digit repeated

Formula used:

Number of arrangements of n things taken r at a time = P(n, r)

$P(n,r) = \frac{n!}{(n-r)!}$

∴ The total number of ways

= the number of arrangements of 5 things taken 4 at a time

= P(5, 4)

$= \frac{5!}{(5-4)!}$

$= \frac{5!}{1!}$

$= \frac{5 \times 4 \times 3 \times 2 \times 1}{1}$

= 120

Hence, total number of 4-digit numbers using digits 1 to 5 with no digit repeated are 120

Now, for 4-digit even number from digits 1, 2, 3, 4 and 5:

Let 4-digit even number be

x	y	z	t

Fix the position of unit's place i.e. t as an even number for which we have 2 choices (2 or 4)

Now, for position x we have remaining 4 choices

Similarly, for position y and z we have 3 and 2 choices respectively

Total number of even numbers are

= multiplication of choices of x y z t

= 4 × 3 × 2 × 2

= 48

Hence, total number of 4-digit numbers using digits 1 to 5 with no digit repeated are 48

32. Question

All the letters of the word ‘EAMCOT’ are arranged in different possible ways. Find the number of arrangements in which no two vowels are adjacent to each other.

Answer

Given, word is ‘EAMCOT’

To find: number of arrangements in which no two vowels are adjacent to each other

Vowels in word ‘EAMCOT’ = 3(E, A, O) and consonants = 3(M, C, T)

Let vowels be denoted by V

Now, fix the position by Vowels like this:

	V		V		V	

The remaining 4 places can be occupied by 3 consonants

Now, arrange 3 consonants at 4 places and 3 vowels at 3 places

Formula used:

Number of arrangements of n things taken all at a time = P(n, n)

$$P(n,r) = \frac{n!}{(n-r)!}$$

∴ Total number of arrangements of vowels

= the number of arrangements of 3 things taken all at a time

= P(3, 3)

$$= \frac{3!}{(3-3)!}$$

$$= \frac{3!}{0!}$$

{∵ 0! = 1}

= 3!

= 3 × 2 × 1

= 6

Formula used:

Number of arrangements of n things taken r at a time = P(n, r)

$$P(n,r) = \frac{n!}{(n-r)!}$$

∴ Total number of arrangements of consonants

= the number of arrangements of 4 things taken 3 at a time

$= P(4, 3)$

$= \frac{4!}{(4-3)!}$

$= \frac{4!}{1!}$

$= 4!$

$= 4 \times 3 \times 2 \times 1$

$= 24$

Hence, total number of arrangements in which no two vowels are adjacent to each other, 6 × 24 = 144

Exercise 16.4

1. Question

In how many ways can the letters of the word 'FAILURE' be arranged so that the consonants may occupy only odd positions?

Answer

Given: the word is 'FAILURE.'

To find: number of arrangements so that the consonants occupy only odd positions

Number of vowels in word 'FAILURE' = 4(E, A, I, U)

Number of consonants = 3(F, L, R)

Let consonants be denoted by C

Odd positions are 1, 3, 5 or 7

Now, fix the position of consonants like this:

C		C		C		C

So, arrange these 3 consonants at 4 places

Formula used:

Number of arrangements of n things taken r at a time = P(n, r)

$$P(n,r) = \frac{n!}{(n-r)!}$$

∴ Total number of arrangements of consonants

= the number of arrangements of 4 things taken 3 at a time

$= P(4, 3)$

$= \frac{4!}{(4-3)!}$

$= \frac{4!}{1!}$

$= 4!$

$= 4 \times 3 \times 2 \times 1$

$= 24$

Remaining 3 even places and 1 odd place can be occupied by 4 vowels

So, arrange these vowels at remaining places

Formula used:

Number of arrangements of n things taken all at a time = P(n, n)

$$P(n,r) = \frac{n!}{(n-r)!}$$

∴ Total number of arrangements of vowels

= the number of arrangements of 4 things taken all at a time

= P(4, 4)

$$= \frac{4!}{(4-4)!}$$

$$= \frac{4!}{0!}$$

{∵ 0! = 1}

= 4!

= 4 × 3 × 2 × 1

= 24

Hence, the number of arrangements so that the consonants occupy only odd positions = 24 × 24 = 576

2 A. Question

In how many ways can the letters of the word ‘STRANGE’ be arranged so that

the vowels come together?

Answer

Given: the word is ‘STRANGE.’

To find: a number of arrangements in which vowels come together

Number of vowels in this word = 2(A, E)

Now, consider these two vowels as one entity(AE together as a single letter)

So, the total number of letters = 6(AE S T R N G)

Formula used:

Number of arrangements of n things taken all at a time = P(n, n)

$$P(n,r) = \frac{n!}{(n-r)!}$$

∴ Total number of arrangements

= the number of arrangements of 6 things taken all at a time

= P(6, 6)

$$= \frac{6!}{(6-6)!}$$

$$= \frac{6!}{0!}$$

{∵ 0! = 1}

= 6!

= 6 × 5 × 4 × 3 × 2 × 1

= 720

Two vowels which are together as a letter can be arranged in 2

Ways like EA or AE

Hence, total number of arrangements in which vowels come together = 2 × 720 = 1440

2 B. Question

In how many ways can the letters of the word 'STRANGE' be arranged so that

the vowels never come together?

Answer

Given: the word is 'STRANGE.'

To find: a number of arrangements in which vowels never come together

To find out these, we will find the total number of arrangements irrespective of any condition and subtract those arrangements in which vowels come together

Total number of letters in the word = 7

Formula used:

Number of arrangements of n things taken all at a time = P(n, n)

$$P(n,r) = \frac{n!}{(n-r)!}$$

∴ Total number of arrangements irrespective of any condition

= the number of arrangements of 7 things taken all at a time

= P(7, 7)

$$= \frac{7!}{(7-7)!}$$

$$= \frac{7!}{0!}$$

{∵ 0! = 1}

= 7!

= 7 × 6 × 5 × 4 × 3 × 2 × 1

= 5040

Number of vowels in this word = 2(A, E)

Now, consider these two vowels as one entity(AE together as a single letter)

So, the total number of letters = 6(AE S T R N G)

Formula used:

Number of arrangements of n things taken all at a time = P(n, n)

$$P(n,r) = \frac{n!}{(n-r)!}$$

∴ Total number of arrangements

= the number of arrangements of 6 things taken all at a time

= P(6, 6)

$= \frac{6!}{(6-6)!}$

$= \frac{6!}{0!}$

$\{\because 0! = 1\}$

= 6!

= 6 × 5 × 4 × 3 × 2 × 1

= 720

Two vowels which are together as a letter can be arranged in 2

Ways like EA or AE

∴ Total number of arrangements in which vowels come together = 2 × 720 = 1440

Hence, the total number of arrangements in which vowel never come together = 5040 – 1440 = 3600

2 C. Question

In how many ways can the letters of the word 'STRANGE' be arranged so that

the vowels occupy only the odd places?

Answer

Given: the word is 'STRANGE.'

To find: number of arrangements so that the vowels occupy only odd positions

Number of vowels in word 'STRANGE' = 2(E, A)

Number of consonants = 5(S, T, R, N, G)

Let vowels be denoted by V

Odd positions are 1, 3, 5 or 7

So, fix the position by Vowels like this:

V		V		V		V

Now, arrange these 2 vowels at 4 odd places

Formula used:

Number of arrangements of n things taken r at a time = P(n, r)

$P(n, r) = \frac{n!}{(n-r)!}$

∴ Total number of arrangements of vowels

= the number of arrangements of 4 things taken 2 at a time

= P(4, 2)

$= \frac{4!}{(4-2)!}$

$= \frac{4 \times 3 \times 2!}{2!}$

= 4 × 3

= 12

The remaining 3 even places and 2 odd places can be occupied by 5 consonants

So, arrange these consonants at these places

Formula used:

Number of arrangements of n things taken all at a time = P(n, n)

$$P(n,r) = \frac{n!}{(n-r)!}$$

∴ Total number of arrangements of consonants

= the number of arrangements of 5 things taken all at a time

= P(5, 5)

$$= \frac{5!}{(5-5)!}$$

$$= \frac{5!}{0!}$$

∵ 0! = 1}

= 5!

= 5 × 4 × 3 × 2 × 1

= 120

Hence, the number of arrangements so that the vowels occupy only odd positions = 12 × 120 = 1440

3. Question

How many words can be formed from the letters of the word 'SUNDAY'? How many of these begin with D?

Answer

Given, the word is 'SUNDAY.'

To find: Number of words that can be formed using letters of the given word and number of words starting with D

Total number of letters in the word = 6

So, arrange these 6 letters

Formula used:

Number of arrangements of n things taken all at a time = P(n, n)

$$P(n,r) = \frac{n!}{(n-r)!}$$

∴ Total number of arrangements

= the number of arrangements of 6 things taken all at a time

= P(6, 6)

$$= \frac{6!}{(6-6)!}$$

$$= \frac{6!}{0!}$$

{∵ 0! = 1}

= 6!

$= 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$= 720$

Hence, the total number of words can be made by letters of the word ‘SUNDAY’ = 720

Now, we need to find out a number of words starting with D

So, fix the position of first letter as D:

D					

Remaining number of letters = 5

Now, arrange these 5 letters at 5 places.

Formula used:

Number of arrangements of n things taken all at a time = P(n, n)

$$P(n, r) = \frac{n!}{(n-r)!}$$

∴ The total number of ways

= the number of arrangements of 5 things taken all at a time

= P(5, 5)

$$= \frac{5!}{(5-5)!}$$

$$= \frac{5!}{0!}$$

{∵ 0! = 1}

= 5!

$= 5 \times 4 \times 3 \times 2 \times 1$

= 120

Hence, the possible number of words using letters of ‘SUNDAY’ starting with ‘D’ is 120

4. Question

How many words can be formed out of the letters of the word, ‘ORIENTAL,’ so that the vowels always occupy the odd places?

Answer

Given: the word is ‘ORIENTAL.’

To find: number of arrangements so that the vowels occupy only odd positions

Number of vowels in the word ‘ORIENTAL’ = 4(O, I, E, A)

Number of consonants in given word = 4(R, N, T, L)

Let vowels be denoted by V

Odd positions are 1, 3, 5 or 7

So, fix the position by vowels like this:

V		V		V		V	

Now, arrange these 4 vowels at 4 places

Formula used:

Number of arrangements of n things taken all at a time = P(n, n)

$$P(n,r) = \frac{n!}{(n-r)!}$$

∴ Total number of arrangements of vowels

= the number of arrangements of 4 things taken all at a time

= P(4, 4)

$$= \frac{4!}{(4-4)!}$$

$$= \frac{4!}{0!}$$

{∵ 0! = 1}

= 4!

= 4 × 3 × 2 × 1

= 24

The remaining 4 even places can be occupied by 4 consonants

So, arrange 4 consonants at remaining places

Formula used:

Number of arrangements of n things taken all at a time = P(n, n)

$$P(n,r) = \frac{n!}{(n-r)!}$$

∴ Total number of arrangements of consonants

= the number of arrangements of 4 things taken all at a time

= P(4, 4)

$$= \frac{4!}{(4-4)!}$$

$$= \frac{4!}{0!}$$

{∵ 0! = 1}

= 4!

= 4 × 3 × 2 × 1

= 24

Hence, the number of arrangements so that the vowels occupy only odd positions = 24 × 24 = 576

5. Question

How many different words can be formed with the letters of word ‘SUNDAY’? How many of the words begin with N? How many begin with N and end in Y?

Answer

Given: the word is ‘SUNDAY.’

To find: number of words that can be formed with the letters of the given word, that can begin with N, and that can begin with N and end in Y

Total number of letters = 6

Formula used:

Number of arrangements of n things taken all at a time = P(n, n)

$$P(n,r) = \frac{n!}{(n-r)!}$$

∴ Total number of arrangements

= the number of arrangements of 6 things taken all at a time

= P(6, 6)

$$= \frac{6!}{(6-6)!}$$

$$= \frac{6!}{0!}$$

{∵ 0! = 1}

= 6!

= 6 × 5 × 4 × 3 × 2 × 1

= 720

Hence, the total number of words can be made by letters of the word 'SUNDAY' = 720

Now, we need to find out of a number of words starting with N

So, fix the position of the first letter as N:

N					

Remaining number of letters in the word 'SUNDAY' = 5

Now, we need to arrange these 5 letters at 5 places.

Formula used:

Number of arrangements of n things taken all at a time = P(n, n)

$$P(n,r) = \frac{n!}{(n-r)!}$$

∴ The total number of ways

= the number of arrangements of 5 things taken all at a time

= P(5, 5)

$$= \frac{5!}{(5-5)!}$$

$$= \frac{5!}{0!}$$

{∵ 0! = 1}

= 5!

= 5 × 4 × 3 × 2 × 1

= 120

Hence, the possible number of words using letters of 'SUNDAY' starting with 'N' is 120

Now, we need to find out a number of words starting with N and ending with Y

So, fix the position of first and last letter as N and Y:

N					Y

Remaining number of letters = 4

Now, we need to arrange these 4 letters at 4 places.

Formula used:

Number of arrangements of n things taken all at a time = P(n, n)

$$P(n,r) = \frac{n!}{(n-r)!}$$

∴ The total number of ways

= the number of arrangements of 4 things taken all at a time

= P(4, 4)

$$= \frac{4!}{(4-4)!}$$

$$= \frac{4!}{0!}$$

{∵ 0! = 1}

= 4!

= 4 × 3 × 2 × 1

= 24

Hence, the possible number of words using letters of 'SUNDAY' starting with 'N' and ending with 'Y' are 24

6 A. Question

How many different words can be formed from the letters of the word 'GANESHPURI'? In how many of these words:

the letter G always occupies the first place?

Answer

Given: the word is 'GANESHPURI.'

To find: number of words starting with G

So, fix the position of the first letter as G

G									

Remaining number of letters in the word = 9

Now, we need to arrange these 9 letters at 9 places.

Formula used:

Number of arrangements of n things taken all at a time = P(n, n)

$$P(n,r) = \frac{n!}{(n-r)!}$$

∴ The total number of ways

= the number of arrangements of 9 things taken all at a time

= P(9, 9)

$$= \frac{9!}{(9-9)!}$$

$$= \frac{9!}{0!}$$

$\{\because 0! = 1\}$

$= 9!$

Hence, a possible number of words using letters of ‘GANESHPURI’ starting with ‘G’ is 9!

6 B. Question

How many different words can be formed from the letters of the word ‘GANESHPURI’? In how many of these words:

the letter P and I respectively occupy the first and last place?

Answer

Given: the word is ‘GANESHPURI.’

To find: number of words starting with P and ending with I

So, fix the position of first and last letter as P and I

G									I

Remaining number of letters in the word = 8

Now, we need to arrange these 8 letters at 8 places.

Formula used:

Number of arrangements of n things taken all at a time = P(n, n)

$$P(n,r) = \frac{n!}{(n-r)!}$$

∴ The total number of ways

= the number of arrangements of 8 things taken all at a time

= P(8, 8)

$$= \frac{8!}{(8-8)!}$$

$$= \frac{8!}{0!}$$

$\{\because 0! = 1\}$

$= 8!$

Hence, a possible number of words using letters of ‘GANESHPURI’ starting with ‘P’ and ending with ‘I’ are 8!

6 C. Question

How many different words can be formed from the letters of the word ‘GANESHPURI’? In how many of these words:

Are the vowels always together?

Answer

Given: the word is ‘GANESHPURI.’

To find: number of words in which vowels are always together

Number of vowels in this word = 4(A, E, I, U)

Now, consider these four vowels as one entity(AEIU together as a single letter) and arrange these letters

So, the total number of letters = 7(AEIU G N S H P R)

Formula used:

Number of arrangements of n things taken all at a time = P(n, n)

$$P(n,r) = \frac{n!}{(n-r)!}$$

∴ Total number of arrangements

= the number of arrangements of 7 things taken all at a time

= P(7, 7)

$$= \frac{7!}{(7-7)!}$$

$$= \frac{7!}{0!}$$

{∵ 0! = 1}

= 7!

= 7 × 6 × 5 × 4 × 3 × 2 × 1

= 5040

Now, 4 vowels which are together as a letter can be arranged in 4! (like EAIU or AEUI)

= 4 × 3 × 2 × 1 = 24 ways

∴ Total number of arrangements in which vowels come together = 24 × 5040 = 120960

6 D. Question

How many different words can be formed from the letters of the word 'GANESHPURI'? In how many of these words:

the vowels always occupy even places?

Answer

Given: the word is 'GANESHPURI.'

To find: number of arrangements so that the vowels occupy only even positions

Number of vowels in the word 'GANESHPURI' = 4(A, E, I, U)

Number of consonants = 6(G, N, S, H, R, I)

Let a vowel be denoted by V

Even positions are 2, 4, 6, 8 or 10

Now, fix the position by Vowels like this:

	V		V		V		V		V

Now, arrange 4 vowels at these 5 places

Formula used:

Number of arrangements of n things taken r at a time = P(n, r)

$$P(n,r) = \frac{n!}{(n-r)!}$$

Total number of arrangements of vowels

= the number of arrangements of 5 things taken 4 at a time

= P(5, 4)

$$= \frac{5!}{(5-4)!}$$

$$= \frac{5!}{1!}$$

= 5!

= 5 × 4 × 3 × 2 × 1

= 120

The remaining 1 even place and 5 odd places can be occupied by 6 consonants

So, arrange 6 consonants at these remaining 6 places

Formula used:

Number of arrangements of n things taken all at a time = P(n, n)

$$P(n, r) = \frac{n!}{(n-r)!}$$

∴ Total number of arrangements of consonants

= the number of arrangements of 6 things taken all at a time

= P(6, 6)

$$= \frac{6!}{(6-6)!}$$

$$= \frac{6!}{0!}$$

{∵ 0! = 1}

= 6!

= 6 × 5 × 4 × 3 × 2 × 1

= 720

Hence, number of arrangements so that the vowels occupy only even positions = 120 × 720 = 86400

7 A. Question

How many permutations can be formed by the letters of the word, ‘VOWELS,’ when

there is no restriction on letters?

Answer

Given: the word is ‘VOWELS.’

To find: number of words that can be formed using letters of the given word

Total number of letters in the word = 6

Formula used:

Number of arrangements of n things taken all at a time = P(n, n)

$$P(n, r) = \frac{n!}{(n-r)!}$$

∴ Total number of arrangements

= the number of arrangements of 6 things taken all at a time

= P(6, 6)

$$= \frac{6!}{(6-6)!}$$

$$= \frac{6!}{0!}$$

{∵ 0! = 1}

= 6!

= 6 × 5 × 4 × 3 × 2 × 1

= 720

Hence, the total number of words can be made by letters of the word 'VOWELS' = 720

7 B. Question

How many permutations can be formed by the letters of the word, 'VOWELS,' when

each word begins with E?

Answer

Given: the word is 'VOWELS.'

To find: number of words using letters of the given word starting with E

So, fix the position of the first letter as E:

E					

Remaining number of letters = 5

Now, we need to arrange these 5 letters at the remaining 5 places.

Formula used:

Number of arrangements of n things taken all at a time = P(n, n)

$$P(n, r) = \frac{n!}{(n-r)!}$$

∴ The total number of ways

= the number of arrangements of 5 things taken all at a time

= P(5, 5)

$$= \frac{5!}{(5-5)!}$$

$$= \frac{5!}{0!}$$

{∵ 0! = 1}

= 5!

= 5 × 4 × 3 × 2 × 1

= 120

Hence, the possible number of words using letters of 'VOWELS' starting with 'E' is 120

7 C. Question

How many permutations can be formed by the letters of the word, 'VOWELS,' when

ach word begins with O and ends with L?

Answer

Given: the word is 'VOWELS.'

To find: number of words using letters of the given word starting with O and ending with L

So, fix the position of first and last letter as O and L:

O					L

Remaining number of letters = 4

Now, we need to arrange these 4 letters at the remaining 4 places.

Formula used:

Number of arrangements of n things taken all at a time = P(n, n)

$$P(n,r) = \frac{n!}{(n-r)!}$$

∴ The total number of ways

= the number of arrangements of 4 things taken all at a time

= P(4, 4)

$$= \frac{4!}{(4-4)!}$$

$$= \frac{4!}{0!}$$

{∵ 0! = 1}

= 4!

= 4 × 3 × 2 × 1

= 24

Hence, the possible number of words using letters of ' VOWELS' starting with 'O' and ending with 'L' is 24

7 D. Question

How many permutations can be formed by the letters of the word, 'VOWELS,' when

all vowels come together?

Answer

Given: the word is 'VOWELS.'

To find: number of words in which vowels always come together

Number of vowels in this word = 2(O, E)

Now, consider these two vowels as one entity(OE together as a single letter)

So, the total number of letters = 5 (OE V W L S)

Formula used:

Number of arrangements of n things taken all at a time = P(n, n)

$$P(n, r) = \frac{n!}{(n-r)!}$$

∴ Total number of arrangements

= the number of arrangements of 5 things taken all at a time

= P(5, 5)

$$= \frac{5!}{(5-5)!}$$

$$= \frac{5!}{0!}$$

{∵ 0! = 1}

= 5!

= 5 × 4 × 3 × 2 × 1

= 120

Now, 2 vowels which are together as a letter can be arranged in 2! (like OE or EO)

= 2 × 1 = 2 ways

Total number of words in which vowels come together = 2 × 120 = 240

7 E. Question

How many permutations can be formed by the letters of the word, 'VOWELS,' when all consonants come together?

Answer

Given: the word is 'VOWELS.'

To find: number of words in which consonants always come together

Number of consonants in this word = 4(V, W, L, S)

Now, consider these four consonants as one entity(VWLS together as a single letter)

So, the total number of letters = 3 (VWLS O E)

Formula used:

Number of arrangements of n things taken all at a time = P(n, n)

$$P(n, r) = \frac{n!}{(n-r)!}$$

∴ Total number of arrangements

= the number of arrangements of 3 things taken all at a time

= P(3, 3)

$$= \frac{3!}{(3-3)!}$$

$$= \frac{3!}{0!}$$

{∵ 0! = 1}

= 3!

= 3 × 2 × 1

= 6

Now, 4 consonants which are together as a letter can be arranged in 4! (like WLVS or SWLV)

= 4 × 3 × 2 × 1 = 24 ways

Total number of words in which vowels come together = 24 × 6 = 144

8. Question

How many words can be formed out of the letters of the word 'ARTICLE,' so that vowels occupy even places?

Answer

Given: the word is 'ARTICLE.'

To find: number of arrangements so that the vowels occupy only even positions

Number of vowels in word 'ARTICLE' = 3(A, E, I)

Number of consonants = 4(R, T, C, L)

Let vowel be denoted by V

Even positions will be 2, 4 or 6

So, fix the position by Vowels like this:

	V		V		V	

Now, arrange 3 vowels at 3 places

Formula used:

Number of arrangements of n things taken r at a time = P(n, r)

$$P(n,r) = \frac{n!}{(n-r)!}$$

∴ Total number of arrangements of vowels

= the number of arrangements of 3 things taken 3 at a time

= P(3, 3)

$$= \frac{3!}{(3-3)!}$$

$$= \frac{3!}{0!}$$

{∵ 0! = 1}

= 3!

= 3 × 2 × 1

= 6

The remaining 4 odd places can be occupied by 4 consonants

So, arrange 4 consonants at remaining 4 places

Formula used:

Number of arrangements of n things taken all at a time = P(n, n)

$$P(n,r) = \frac{n!}{(n-r)!}$$

∴ Total number of arrangements of consonants

= the number of arrangements of 4 things taken all at a time

= P(4, 4)

$= \frac{4!}{(4-4)!}$

$= \frac{4!}{0!}$

{∵ 0! = 1}

= 4!

= 4 × 3 × 2 × 1

= 24

Hence, the number of arrangements so that the vowels occupy only even positions = 6 × 24 = 144

9. Question

In how many ways can lawn tennis mixed double be made up from seven married couples if no husband and wife play in the same set?

Answer

Given: there are 7 married couples

∴ Number of women and men = 7 each

To find: number of ways of selecting 2 men (A, B) and 2 women(C, D) such that C and D are not wives of A and B

Firstly, select 2 men out of a total of 7 men

Formula used:

Number of ways of selecting n things taken r at a time = C(n, r)

$C(n,r) = \frac{n!}{(n-r)!\,r!}$

∴ Number of ways of selecting 7 things taken 2 at a time

= C(7, 2)

$= \frac{7!}{(7-2)!\,2!}$

$= \frac{7!}{5!\,2!}$

$= \frac{7 \times 6 \times 5!}{5! \times 2}$

= 7 × 3

= 21

Now, select 2 women out of 5 men(excluding wives of A and B)

Formula used:

Number of ways of selecting n things taken r at a time = C(n, r)

$C(n,r) = \frac{n!}{(n-r)!\,r!}$

∴ Number of ways of selecting 5 things taken 2 at a time

= C(5, 2)

$= \frac{5!}{(5-2)!\,2!}$

$= \frac{5!}{3!\,2!}$

$= \frac{5 \times 4 \times 3!}{3! \times 2}$

= 5 × 2

= 10

Numbers of ways in which 4 members of the team can be chosen = 21 × 10

= 210

Now, A can choose B as team partner (⇒ C's partner is D) or

A can choose C as team partner (⇒ B's partner is D) or

A can choose D as team partner (⇒ C's partner is B)

⇒ The team can be formed in these 3 possible ways

∴ Number of possible ways in which lawn tennis mixed double be made up from seven married couples if no husband and wife play in the same set = 210 × 3 = 630

10. Question

m men and n women are to be seated in a row so that no two women sit together. If m>n then show that the number of ways in which they can be seated as $\frac{m!(m+1)!}{(m-n+1)!}$.

Answer

Given: there are m men and n women where m>n

To find: out their possible way of sitting arrangements in a row such that no two women sit together

Let m = 2 then possible arrangement is

_ m _ m _

Here 3(2 + 1) gaps for women can by made by 2 men so that no two of them comes together

∴ When m men are there, m seats can be occupied by men and m + 1 seat can by women

Formula used:

Number of arrangements of n things taken all at a time = P(n, n)

$P(n,r) = \frac{n!}{(n-r)!}$

∴ Total number of arrangements of men

= the number of arrangements of m things taken all at a time

= P(m, m)

$= \frac{m!}{(m-m)!}$

$= \frac{m!}{0!}$

$\{\because 0! = 1\}$

$= m!$

Formula used:

Number of arrangements of n things taken r at a time = P(n, r)

$$P(n, r) = \frac{n!}{(n-r)!}$$

∴ Total number of arrangements of women

= the number of arrangements of m + 1 things taken n at a time

= P(m + 1, n)

$$= \frac{(m+1)!}{(m+1-n)!}$$

$$= \frac{(m+1)!}{(m-n+1)!}$$

Hence, total possible arrangements of m men and n women in a row such that no two women come together

$$= m! \times \frac{(m+1)!}{(m-n+1)!}$$

$$= \frac{m!\,(m+1)!}{(m-n+1)!}$$

Hence, Proved.

11 A. Question

How many words (with or without dictionary meaning) can be made from the letters in the word MONDAY, assuming that no letter is repeated, if

4 letters are used at a time?

Answer

Given: the word is ‘MONDAY.’

Total number of letters in the word = 6

To find: possible number of four-letter words can be made using the letters of the word ‘MONDAY.’

Formula used:

Number of arrangements of n things taken r at a time = P(n, r)

$$P(n, r) = \frac{n!}{(n-r)!}$$

∴ Total number of arrangements

= the number of arrangements of 6 things taken 4 at a time

= P(6, 4)

$$= \frac{6!}{(6-4)!}$$

$$= \frac{6!}{2!}$$

$$= \frac{6 \times 5 \times 4 \times 3 \times 2!}{2!}$$

$= 6 \times 5 \times 4 \times 3$

$= 360$

Hence, the total number of four-letter words can be made by letters of the word ‘MONDAY’ = 360

11 B. Question

How many words (with or without dictionary meaning) can be made from the letters in the word MONDAY, assuming that no letter is repeated, if

Are all letters used at a time?

Answer

Given: the word is ‘MONDAY.’

To find: possible number of words using all the letters of the word ‘MONDAY.’

Total number of letters in the word= 6

Formula used:

Number of arrangements of n things taken all at a time = P(n, n)

$$P(n,r) = \frac{n!}{(n-r)!}$$

∴ Total number of arrangements

= the number of arrangements of 6 things taken all at a time

= P(6, 6)

$$= \frac{6!}{(6-6)!}$$

$$= \frac{6!}{0!}$$

$\{\because 0! = 1\}$

$= 6!$

$= 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$= 720$

Hence, the total number of words can be made by letters of the word ‘MONDAY’ = 720

11 C. Question

How many words (with or without dictionary meaning) can be made from the letters in the word MONDAY, assuming that no letter is repeated, if

all letters are used but first is a vowel?

Answer

Given: the word is ‘MONDAY.’

To find: possible number of words using all the letters of the word ‘MONDAY’ but the first letter is a vowel

Total number of vowels = 2(A, O)

Now, fix the position of 1 vowel out of these two at first place which can be done in 2 ways.

A					

or

O					

Now, we need to fill the remaining 5 places

Remaining places for letters = 5

So, arrange 5 letters at 5 places

Formula used:

Number of arrangements of n things taken all at a time = P(n, n)

$$P(n,r) = \frac{n!}{(n-r)!}$$

∴ Total number of arrangements

= the number of arrangements of 5 things taken all at a time

= P(5, 5)

$$= \frac{5!}{(5-5)!}$$

$$= \frac{5!}{0!}$$

{∵ 0! = 1}

= 5!

= 5 × 4 × 3 × 2 × 1

= 120

Hence, the total number of words can be made by using all letters of the word 'MONDAY,' but the first letter is vowel = 120 × 2 = 240

12. Question

How many three letter words can be made using the letters of the word 'ORIENTAL.'

Answer

Given: the word is 'ORIENTAL.'

Total number of letters = 8

To find: possible number of three-letter words can be made using the letters of the word 'ORIENTAL.'

Formula used:

Number of arrangements of n things taken r at a time = P(n, r)

$$P(n,r) = \frac{n!}{(n-r)!}$$

∴ Total number of arrangements

= the number of arrangements of 8 things taken 3 at a time

= P(8, 3)

$$= \frac{8!}{(8-3)!}$$

$$= \frac{8!}{5!}$$

$= \frac{8 \times 7 \times 6 \times 5!}{5!}$

= 8 × 7 × 6

= 336

Hence, the total number of three-letter words can be made by letters of the word 'ORIENTAL' = 336

Exercise 16.5

1 A. Question

Find the number of words formed by permuting all the letters of the following words :

INDEPENDENCE

Answer

Since we know, Permutation of n objects taking r at a time is ^nP_r,and permutation of n objects taking all at a time is n!

And, we also know Permutation of n objects taking all at a time having p objects of the same type, q objects of another type, r objects of another type is $\frac{n!}{p! \times q! \times r!}$. i.e. the, number of repeated objects of same type are in denominator multiplication with factorial.

Given, the word INDEPENDENCE. It has 12 letters and it has 3 repeated letters 'N', 'D,' 'E.' Of which, the letter N is repeated thrice, the letter D is repeated twice, and the letter E is repeated 4 times in that word. All other letters are distinct.

The problem can now be rephrased as to find total number of permutations of 12 objects (I, N, D, E, P, E, N, D, E, N, C, E) of which three objects are of same type (N, N, N), two objects are of another type (D, D), and four objects are of different type (E, E, E, E).

Total number of such permutations $= \frac{12!}{3! \times 2! \times 4!}$

$= \frac{5 \times 6 \times 7 \times 8 \times 9 \times 10 \times 11 \times 12}{(3 \times 2 \times 1) \times (2 \times 1)}$

= 1663200

Hence, a total number of permutations of the word INDEPENDENCE is 1663200.

1 B. Question

Find the number of words formed by permuting all the letters of the following words :

INTERMEDIATE

Answer

Since we know, Permutation of n objects taking r at a time is ^nP_r,and permutation of n objects taking all at a time is n!

And, we also know Permutation of n objects taking all at a time having p objects of the same type, q objects of another type, r objects of another type is $\frac{n!}{p! \times q! \times r!}$. i.e. the, number of repeated objects of same type are in denominator multiplication with factorial.

Given, the word INTERMEDIATE. It has 12 letters, and it has 3 repeated letters 'I,' 'T,' 'E.' Of which, the letter I is repeated twice, the letter T is repeated twice, and the letter E is repeated 3 times in that word. All other letters are distinct.

The problem can now be rephrased as to find total number of permutations of 12 objects (I, N, T, E, R, M, E, D, I, A, T, E) of which two objects are of same type (I, I), two objects are of another type (T, T), and three objects are of different type (E, E, E).

Total number of such permutations $= \frac{12!}{2! \times 2! \times 3!}$

$$= \frac{4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10 \times 11 \times 12}{(2 \times 1) \times (2 \times 1)}$$

= 19958400

Hence, a total number of permutations of the word INTERMEDIATE is 19958400.

1 C. Question

Find the number of words formed by permuting all the letters of the following words :

ARRANGE

Answer

Since we know, Permutation of n objects taking r at a time is ^nP_r,and permutation of n objects taking all at a time is n!

And, we also know Permutation of n objects taking all at a time having p objects of the same type, q objects of another type, r objects of another type is $\frac{n!}{p!\times q!\times r!}$. i.e. the, number of repeated objects of same type are in denominator multiplication with factorial.

Given, the word ARRANGE. It has 7 letters, and it has 2 repeated letters ‘A’, ‘R.’ Of which, the letter A is repeated twice, and the letter R is also repeated twice. All other letters are distinct.

The problem can now be rephrased as to find a total number of permutations of 7 objects (A, R, R, A, N, G, E) of which two objects are of same type (A, A), and two objects are of another type (R, R).

Total number of such permutations $= \frac{7!}{2!\times 2!}$

$$= \frac{5 \times 6 \times 7 \times 8 \times 9 \times 10 \times 11 \times 12}{(3 \times 2 \times 1) \times (2 \times 1)}$$

= 1260

Hence, a total number of permutations of the word ARRANGE is 1260.

1 D. Question

Find the number of words formed by permuting all the letters of the following words :

INDIA

Answer

Since we know, Permutation of n objects taking r at a time is ^nP_r,and permutation of n objects taking all at a time is n!

And, we also know Permutation of n objects taking all at a time having p objects of the same type, q objects of another type, r objects of another type is $\frac{n!}{p!\times q!\times r!}$. i.e. the, number of repeated objects of same type are in denominator multiplication with factorial.

Given, the word INDIA. It has 5 letters, and it has 1 repeated letter ‘I.’ The letter I is repeated twice and all other letters are distinct.

The problem can now be rephrased as to find a total number of permutations of 5 objects (I, N, D, I, A) of which two objects are of same type (I, I).

Total number of such permutations $= \frac{5!}{2!}$

$$= \frac{1 \times 2 \times 3 \times 4 \times 5}{2 \times 1}$$

= 60

Hence, a total number of permutations of the word INDIA is 60.

1 E. Question

Find the number of words formed by permuting all the letters of the following words :

PAKISTAN

Answer

Since we know, Permutation of n objects taking r at a time is ${}^nP_r$,and permutation of n objects taking all at a time is n!

And, we also know Permutation of n objects taking all at a time having p objects of the same type, q objects of another type, r objects of another type is $\frac{n!}{p!\times q!\times r!}$. i.e. the, number of repeated objects of same type are in denominator multiplication with factorial.

Given, the word PAKISTAN. It has 8 letters, and it has 1 repeated letter 'A.' The letter A is repeated twice, and all other letters are distinct.

The problem can now be rephrased as to find a total number of permutations of 8 objects (P, A, K, I, S, T, A, N) of which two objects are of same type (A, A).

Total number of such permutations $= \frac{8!}{2!}$

$= \frac{1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8}{2 \times 1}$

= 20160

Hence, a total number of permutations of the word PAKISTAN is 20160.

1 F. Question

Find the number of words formed by permuting all the letters of the following words :

RUSSIA

Answer

Since we know, Permutation of n objects taking r at a time is ${}^nP_r$,and permutation of n objects taking all at a time is n!

And, we also know Permutation of n objects taking all at a time having p objects of the same type, q objects of another type, r objects of another type is $\frac{n!}{p!\times q!\times r!}$. i.e. the, number of repeated objects of same type are in denominator multiplication with factorial.

Given, the word RUSSIA. It has 6 letters, and it has 1 repeated letter 'S.' The letter S is repeated twice, and all other letters are distinct.

The problem can now be rephrased as to find a total number of permutations of 6 objects (R, U, S, S, I, A) of which two objects are of same type (S, S).

Total number of such permutations $= \frac{6!}{2!}$

$= \frac{1 \times 2 \times 3 \times 4 \times 5 \times 6}{2 \times 1}$

= 360

Hence, a total number of permutations of the word RUSSIA is 360.

1 G. Question

Find the number of words formed by permuting all the letters of the following words :

SERIES

Answer

Since we know, Permutation of n objects taking r at a time is ${}^nP_r$,and permutation of n objects taking all at a time is n!

And, we also know Permutation of n objects taking all at a time having p objects of the same type, q objects of another type, r objects of another type is $\frac{n!}{p!\times q!\times r!}$. i.e. the, number of repeated objects of same type are in denominator multiplication with factorial.

Given, the word SERIES. It has 6 letters, and it has 2 repeated letter 'S,' and 'E.' The letter S is repeated twice, and letter E is also repeated twice. And all other letters are distinct.

The problem can now be rephrased as to find a total number of permutations of 6 objects (S, E, R, I, E, S) of which two objects are of same type (S, S), and two objects are of another type (E, E).

Total number of such permutations $= \frac{6!}{2!\times 2!}$

$= \frac{1\times 2\times 3\times 4\times 5\times 6}{(2\times 1)\times(2\times 1)}$

= 180

Hence, a total number of permutations of the word SERIES is 180.

1 H. Question

Find the number of words formed by permuting all the letters of the following words :

EXERCISES

Answer

Since we know, Permutation of n objects taking r at a time is ^nP_r,and permutation of n objects taking all at a time is n!

And, we also know Permutation of n objects taking all at a time having p objects of the same type, q objects of another type, r objects of another type is $\frac{n!}{p!\times q!\times r!}$. i.e. the, number of repeated objects of same type are in denominator multiplication with factorial.

Given, the word EXERCISES. It has 9 letters, and it has 2 repeated letters 'E', and 'S.' The letter E is repeated thrice, and letter S is repeated twice. And all other letters are distinct.

The problem can now be rephrased as to find total number of permutations of 9 objects (E, X, E, R, C, I, S, E, S) of which three objects are of same type (E, E, E), and two objects are of another type (S, S).

Total number of such permutations $= \frac{9!}{3!\times 2!}$

$= \frac{1\times 2\times 3\times 4\times 5\times 6\times 7\times 8\times 9}{(3\times 2\times 1)\times(2\times 1)}$

= 30240

Hence, a total number of permutations of the word EXERCISES is 30240.

1 I. Question

Find the number of words formed by permuting all the letters of the following words :

CONSTANTINOPLE

Answer

Since we know, Permutation of n objects taking r at a time is ^nP_r,and permutation of n objects taking all at a time is n!

And, we also know Permutation of n objects taking all at a time having p objects of the same type, q objects of another type, r objects of another type is $\frac{n!}{p!\times q!\times r!}$. i.e. the, number of repeated objects of same type are in denominator multiplication with factorial.

Given, the word CONSTANTINOPLE. It has 14 letters and it has 3 repeated letters 'O', 'N,' and 'T.' The letter O is repeated twice, the letter N is repeated thrice, and letter T is repeated twice. And all other letters are distinct.

The problem can now be rephrased as to find total number of permutations of 14 objects (C, O, N, S, T, A, N, T, I, N, O, P, L, E) of which two objects are of same type (O, O), three objects are of another type (N, N, N), and the two objects are of different type (T, T).

Total number of such permutations $= \frac{14!}{2! \times 3! \times 2!}$

$= \frac{14!}{24}$

= 3632428750

Hence, total number of permutations of the word CONSTANTINOPLE is 3632428750.

2. Question

In how many ways can the letters of the word 'ALGEBRA' be arranged without changing the relative order of the vowels and consonants?

Answer

Given, the word 'ALGEBRA'. It has 7 letters of which 4 of them are consonants (L, G, B, R) and 3 of them are vowels (A, E, A) repeating the vowel A twice.

We have to find a number of words that can be formed without changing the relative order of vowels and consonants, i.e. if a vowel comes before a consonant in word ALGEBRA, it has to be in the same order in all possible words. For example- 'ELGABRA' and 'AGLEBRA' are such two words.

Since we know, Permutation of n objects taking r at a time is ^nP_r,and permutation of n objects taking all at a time is n!

And, we also know Permutation of n objects taking all at a time having p objects of the same type, q objects of another type, r objects of another type is $\frac{n!}{p! \times q! \times r!}$. i.e. the, number of repeated objects of samethe type are in denominator multiplication with factorial.

The consonants in their positions can be arranged in 4! = 24 number of ways, since there is no repeating letter in consonants set (L, G, B, R).

Similarly, the vowels in their positions can be arranged in 3! / 2! = 3 number of ways, since there is one repeated letter in vowel set (A, E, A), i.e. the letter A repeating twice.

Now, total number of arrangements where relative position of consonants and vowels are not changed will be exactly equal to number of ways we can select one to one element from consonant set to vowel set using Multiplication principle (If an event A can occur in m different ways and another event B can occur in n different ways then a total number of ways of simultaneous occurrence of both events in definite order is m x n.)

i.e. Total arrangements = 24 x 3

= 72

Hence, a total number of ways the word 'ALGEBRA' be arranged such that the relative position of vowels and consonants are unchanged is equaled to 72.

3. Question

How many words can be formed with the letters of the word 'UNIVERSITY,' the vowels remaining together?

Answer

Given, the word UNIVERSITY. It has 4 vowels and 6 consonants. Vowels included in given word are (U, I, E, I) and consonants included in given word are (N, V, R, S, T, Y). In given word, one vowel letter I is repeated twice.

We have to find a number of words that can be formed in such a way that all vowels must come together. For example- NUIEIVRSTY, UIIENRSYTV, YTSRIIEUVN, and RTSEIUIYNV are few words among them. Notice that all vowel letters come in the bunch (In bold letters).

A specific method is usually used for solving such type of problems. According to that, we assume the group of letters that remain together (here U, I, E, I) are assumed to be a single letter and all other letters are as usual counted as a single letter. Now find a number of ways as usual; the number of ways of arranging r letters from a group of n letters is equals to ${}^nP_r$. And the final answer is then multiplied by a number of ways we can arrange the letters in that group which has to be stuck together in it (Here U, I, E, I).

Since we know, Permutation of n objects taking r at a time is ${}^nP_r$,and permutation of n objects taking all at a time is n!

And, we also know Permutation of n objects taking all at a time having p objects of the same type, q objects of another type, r objects of another type is $\frac{n!}{p!\times q!\times r!}$. i.e. the, number of repeated objects of same type are in denominator multiplication with factorial.

Now,

Letters of word UNIVERSITY are U, N, I, V, E, R, S, I, T, Y. (10 letters)

Now from our method, letters are UIEI, N, V, R, S, T, Y. (7 letters)

A total number of arrangements of 7 letters (here all distinct) is 7!

And the total number of arrangements of grouped letters (Here U, I, E, I) is $\frac{4!}{2!}$.

So, our final answer for arranging the letters such that all vowels stick together equals multiplication of 7! and $\frac{4!}{2!}$

Total number of arrangements $= 7! \times \frac{4!}{2!}$

= 60480

Hence, a total number of words formed during the arrangement of letters of word UNIVERSITY such that all vowels remain together is equals to 60480.

4. Question

Find the total number of arrangements of the letters in the expression $a^3\ b^2\ c^4$ when written at full length.

Answer

Given expression $a^3b^2c^4$ i.e. in expansion aaabbcccc.

To find: Number of expressions that can be generated by permuting the letters of given expression aaabbcccc.

Given expression has three repeating characters a, b, and c. The letter a is repeated 3 times, the letter b is repeated 2 times, and the letter c is repeated 4 times.

So, the given problem can now be rephrased as to find a total number of arrangements of 9 objects (3+2+4) of which 3 objects are of the same type, 2 objects are of another type, and 4 objects are of different type.

Since we know, Permutation of n objects taking r at a time is ${}^nP_r$,and permutation of n objects taking all at a time is n!

And, we also know Permutation of n objects taking all at a time having p objects of the same type, q objects of another type, r objects of another type is $\frac{n!}{p!\times q!\times r!}$. i.e. the, number of repeated objects of same type are in denominator multiplication with factorial.

The number of ways of arranging 9 objects of which 3, 2, and 4 objects are of different types is equaled to

$$= \frac{9!}{3! \times 2! \times 4!}$$

= 124

Hence, number of ways of arranging the letters of word/expression aaabbcccc is equals to 124.

5. Question

How many words can be formed with the letters of the word ‘PARALLEL’ so that all L’s do not come together?

Answer

Given, the word PARALLEL. It has 8 letters of which 2 letters (A, L) are repeating. The letter A is repeated twice, and the letter L is repeated thrice in the given word PARALLEL.

To find: Number of ways the letters of word PARALLEL be arranged in such a way that not all L's do come together.

How are we going to find it? First, we find all arrangements of word PARALLEL and then we minus all those arrangements of word PARALLEL in such a way that all L's do come together, from it. This will exactly be the same as- all number of arrangements such that not all L's do come together.

Since we know, Permutation of n objects taking r at a time is ${}^nP_r$,and permutation of n objects taking all at a time is n!

And, we also know Permutation of n objects taking all at a time having p objects of the same type, q objects of another type, r objects of another type is $\frac{n!}{p!\times q!\times r!}$. i.e. the, number of repeated objects of same type are in denominator multiplication with factorial.

A total number of arrangements of word PARALLEL: Total letters 8. Repeating letters A and L, the letter A repeating twice and letter L repeating thrice. The total number of arrangements

$$\Rightarrow \frac{8!}{2! \times 3!}$$

Now we find a total number of arrangements such that all L's do time together.

A specific method is usually used for solving such type of problems. According to that, we assume the group of letters that remain together (here L, L, L) are assumed to be a single letter, and all other letters are as usual counted as a single letter. Now find a number of ways as usual; the number of ways of arranging r letters from a group of n letters is equals to ${}^nP_r$. And the final answer is then multiplied by a number of ways we can arrange the letters in that group which has to be stuck together in it (Here L, L, L).

Letters in word PARALLEL: 8 letters

Letters in a new word: LLL, P, A, A, R, E: 6 letters (Letter A repeated twice).

Total number of word arranging all the letters = $\frac{6!}{2!} \times \frac{3!}{3!}$

where second fraction 3! divided by 3! comes from arranging letters inside the group LLL: arrangements of three letters where all the three letters are same = $\frac{3!}{3!}$ = 1 (Obviously! You can even think of it).

Now, a Total number of arrangements where not all L's do come together is equals to total arrangements of word PARALLEL minus the total number of arrangements in such a way that all L's do come together.

$$\Rightarrow \frac{8!}{2! \times 3!} - \left(\frac{6!}{2!} \times \frac{3!}{3!}\right)$$

= 3000

Hence, a total number of arrangements of word PARALLEL in such a way that not all L's do come together equals to 3000.

6. Question

How many words can be formed by arranging the letters of the word 'MUMBAI' so that all M's come together?

Answer

Given, the word MUMBAI. It has 2 M's, and total 6 letters.

We have to find a number of words that can be formed in such a way that all M's must come together. For example- MMBAIU, AIBMMU, UMMABI are few words among them. Notice that all M letters come in the bunch (In bold letters).

A specific method is usually used for solving such type of problems. According to that, we assume the group of letters that remain together (here M, M) are assumed to be a single letter, and all other letters are as usual counted as a single letter. Now find a number of ways as usual; the number of ways of arranging r letters from a group of n letters is equals to ${}^nP_r$. And the final answer is then multiplied by a number of ways we can arrange the letters in that group which has to be stuck together in it (Here M, M).

Since we know, Permutation of n objects taking r at a time is ${}^nP_r$,and permutation of n objects taking all at a time is n!

And, we also know Permutation of n objects taking all at a time having p objects of same type, q objects of another type, r objects of another type is $\frac{n!}{p!\times q!\times r!}$. i.e. the, number of repeated objects of same type are in denominator multiplication with factorial.

Now,

Letters of word MUMBAI are M, U, M, B, A, I (6 letters)

Now from our method, letters are MM, U, B, A, I. (5 letters)

A total number of arrangements of 5 letters (here all distinct) is 5!

And the total number of arrangements of grouped letters (Here M, M) $= \frac{4!}{2!}$.

So, our final answer for arranging the letters such that all vowels stick together equals multiplication of 5! and $\frac{2!}{2!}$

Total number of arrangements $= 5! \times \frac{2!}{2!}$

= 5!

= 120

Hence, a total number of words formed during the arrangement of letters of word MUMBAI such that all M's remains together equals to 120.

7. Question

How many numbers can be formed with the digits 1, 2, 3, 4, 3, 2, 1 so that the odd digits always occupy the odd places?

Answer

Given, digits 1, 2, 3, 4, 3, 2, 1.

To find: Number of numbers formed from given digits such that odd digits always occupies odd positions/places.

Since we know, Permutation of n objects taking r at a time is ${}^nP_r$,and permutation of n objects taking all at a time is n!

And, we also know Permutation of n objects taking all at a time having p objects of same type, q objects of another type, r objects of another type is $\frac{n!}{p!\times q!\times r!}$. i.e. the, number of repeated objects of same type are in denominator multiplication with factorial.

A total number of ways of arranging 3 even digits = 3! / 2! since there is repetition of a digit (2).

A total number of ways of arranging 4 odd digits = 4! / (2! x 2!) since there are twice repetition of digits (1 and 3).

Total number of ways of arranging the digits such that odd digits always occupies odd places $= \frac{3!}{2!} \times \frac{4!}{2!\times 2!}$

= 18

Hence, the number of ways of arranging the digits such odd digits always occupies odd places is equals to 18.

8. Question

How many different signals can be made from 4 red, 2 white, and 3 green flags by arranging all of them vertically on a flagstaff?

Answer

Given, the flags: Red, White, and Green. Total of 9 flags and it has 3 repeated flags Green, Red, and White. The flag Green is repeated thrice, and flag Red is repeated 4 times, flag White is repeated twice.

To find: Number of ways of arranging the flags vertically on a flagstaff.

The problem can now be rephrased as to find total number of permutations of 9 objects (R, R, R, R, W, W, G, G, G) of which three objects are of same type (G, G, G), and two objects are of another type (W, W), and 4 objects are of different type (R, R, R, R).

Since we know, Permutation of n objects taking r at a time is ^nP_r,and permutation of n objects taking all at a time is n!

And, we also know Permutation of n objects taking all at a time having p objects of the same type, q objects of another type, r objects of another type is $\frac{n!}{p!\times q!\times r!}$. i.e. the, number of repeated objects of same type are in denominator multiplication with factorial.

Total number of such permutations $= \frac{9!}{4!\times 2!\times 3!}$

$= \frac{5\times 6\times 7\times 8\times 9}{(2\times 1)\times(3\times 2\times 1)}$

= 1260

Hence, total number of permutations of the given type is 1260.

9. Question

How many numbers of four digits can be formed with the digits 1, 3, 3, 0?

Answer

Given, the numbers 1, 3, 3, 0. Total of 4 digits, and it has 1 repeated digit 1 repeated twice.

To find: Number of four digit numbers that can be formed using digits 1, 3, 3, 0. Notice that an arrangement in which digit 0 in the first place will not be counted as four digit number. For example- 0331 will not be counted as four digit number since it is a 3 digit number.

The problem can now be rephrased as to find a total number of permutations of 4 objects (1, 3, 3, 0) of which two objects are of same type (1, 1), And all other objects are distinct. But, 0 cannot be in first place (Condition of four digit number).

First, we will find a total number of permutations of these 4 digits and then we will go minus all those permutations in which 0 will come in first place. This will give us exactly number of four-digit numbers that can be formed by permuting the given digits, i.e. 1, 3, 3, 0.

Total number of permutations will be $\frac{4!}{2!}$

Number of permutations in which 0 will come at the first place will be equal to (Number of ways we can arrange the remaining digits, i.e. 1, 3, 3 in the remaining three places) $\frac{3!}{2!}$

Total number of permutations of given digits forming a four-digit number is equal to

$\Rightarrow \frac{4!}{2!} - \frac{3!}{2!}$

$\Rightarrow 12 - 3$

$\Rightarrow 9$

Hence, total number of permutations of 4 digits (1, 3, 3, 0) forming a 4 digit number is 9.

10. Question

In how many ways can the letters of the word ‘ARRANGE’ be arranged so that the two R’s are never together?

Answer

Given, the word ARRANGE. It has 7 letters of which 2 letters (A, R) are repeating. The letter A is repeated twice, and the letter R is also repeated twice in the given word.

To find: Number of ways the letters of word ARRANGE be arranged in such a way that not all R’s do come together.

First, we find all arrangements of word ARRANGE, and then we minus all those arrangements of word ARRANGE in such a way that all R's do come together, from it. This will exactly be the same as- all number of arrangements such that not all R's do come together.

Since we know, Permutation of n objects taking r at a time is ${}^nP_r$,and permutation of n objects taking all at a time is n!

And, we also know Permutation of n objects taking all at a time having p objects of the same type, q objects of another type, r objects of another type is $\frac{n!}{p!\times q!\times r!}$. i.e. the, number of repeated objects of same type are in denominator multiplication with factorial.

A total number of arrangements of word ARRANGE: Total letters 7. Repeating letters A and R, the letter A repeating twice and letter R repeating twice. The total number of arrangements

$$= \frac{7!}{2! \times 2!}$$

Now we find a total number of arrangements such that all R's do come together.

A specific method is usually used for solving such type of problems. According to that, we assume the group of letters that remain together (here R, R) is assumed to be a single letter and all other letters are as usual counted as a single letter. Now find a number of ways as usual; the number of ways of arranging r letters from a group of n letters is equals to ${}^nP_r$. And the final answer is then multiplied by a number of ways we can arrange the letters in that group which has to be stuck together in it (Here R, R).

Letters in word ARRANGE: 7 letters

Letters in a new word: A, RR, A, N, G, E: 6 letters (Letter A repeated twice).

Total number of word arranging all the letters $= \frac{6!}{2!} \times \frac{2!}{2!}$

where second fraction 2! divided by 2! comes from arranging letters inside the group RR: arrangements of two letters where all the two letters are same = 2!/2! = 1 (Obviously! You can even think of it).

Now, a Total number of arrangements where not all R's do come together is equals to total arrangements of word ARRANGE minus the total number of arrangements in such a way that all R's do come together.

$$= \frac{7!}{2! \times 2!} - \left(\frac{6!}{2!} \times \frac{2!}{2!}\right)$$

= 900

Hence, the total number of arrangements of word ARRANGE in such a way that not all R's come together is equals to 900.

11. Question

How many different numbers, greater than 50000 can be formed with the digits 0, 1, 1, 5, 9.

Answer

Given, the numbers 0, 1, 1, 5, 9. Total of 5 digits, and it has 1 repeated digit 1 repeated twice.

To find: Number of numbers that can be formed using digits 0, 1, 1, 5, 9 in such a way that arranged number is greater than 50000. Notice that an arrangement in which the first digit is either 5 or either 9 will only produce number greater than 50000. We have to find such numbers.

The problem can now be rephrased as to find a total number of permutations of 5 objects (0, 1, 1, 5, 9) of which two objects are of same type (1, 1), And all other objects are distinct. But, either 5 or 9 will be only in the first place (According to question).

First, we will find a total number of permutations of these 5 digits starting with 5 and then we will find the total number of permutations of these 5 digits starting with 9. Addition of these two numbers will give us the required numbers greater than 50000.

Total number of permutations starting with 5 will be equals to permutations of remaining digits (0, 1, 1, 9) in 4 remaining places $= \frac{4!}{2!}$

Total number of permutations starting with 9 will be equals to permutations of remaining digits (0, 1, 1, 5) in

4 remaining places = $\frac{4!}{2!}$

Number of permutations in which either 5 or 9 will come at first place will be equal to

$= \frac{4!}{2!} + \frac{4!}{2!}$

= 24

Hence, total number of permutations of 5 digits (0, 1, 1, 5, 9) forming a 5 digit number greater than 50000 is equals to 24.

12. Question

How many words can be formed from the letters of the word 'SERIES' which start with S and end with S?

Answer

Given the word SERIES. A total number of letters in it is 6. Two repeating characters S and E, both repeating twice.

To find: Total number of words that can be formed by permuting all digits in such a way that first place and the last will always be occupied by the letter S. For example- SRIEES, SERIES, SREIES are few words among them. Notice the first and last letter of the word is S.

Total number of such words can be formed by permutation of 4 letters (E, R, I, E) in between two S letters; which will be equals to

$= \frac{4!}{2!}$

= 3 x 4

= 12

The denominator factor of 2! is because there is a repeating letter twice (E) in those 4 letters (E, R, I ,E).

Hence, a total number of words permuting the letters of the word SERIES in such a way that the first and last position is always occupied by the letter S is 12.

13. Question

How many permutations of the letters of the word 'MADHUBANI' do not begin with M but end with I?

Answer

Given, the word MADHUBANI. It has 9 letters of which 1 letter (A) is repeating twice and all other letters of the word is are distinct.

To find: Number of ways the letters of word MADHUBANI be arranged in such a way that the word must not begin with M but ends with I.

Let's assume that I will be at the end of the word we are forming, and will not be changed its position. We will not take it into consideration now as its position is fixed. Now we have 8 letters to arrange.

First we find all arrangements of word MADHUBANI and then we minus all those arrangements of word MADHUBANI in such a way that the word is starting with the letter M. This will exactly be same as- all number of arrangements such that the word will not begins with the letters M and ends with the letter I (As the letter I was already fixed in the last position).

Since we know, Permutation of n objects taking r at a time is ^nP_r,and permutation of n objects taking all at a time is n!

And, we also know Permutation of n objects taking all at a time having p objects of the same type, q objects of another type, r objects of another type is $\frac{n!}{p! \times q! \times r!}$. i.e. the, number of repeated objects of same type are in denominator multiplication with factorial.

A total number of arrangements of word MADHUBANI excluding I: Total letters 8. Repeating letter A, repeating twice. The total number of arrangements will be equals to

$=\frac{8!}{2!}$

Now we find a total number of arrangements such that the word begins with the letter M.

The total number of arrangements of word MADHUBANI excluding I will be equals to permutation of 7 objects (A, D, H, U, B, A, N) taking all together.

Letters : 7

Repeating letter: A (2 times)

Total number of word arranging all the letters $=\frac{7!}{2!}$

Now, a Total number of arrangements where the word not starts with M but ends with I (I was already fixed in the last position) will be equals to total arrangements of word MADHUBAN minus the total number of arrangements in such a way that word starts with letter M.

$=\frac{8!}{2!}-\frac{7!}{2!}$

= 17640

Hence, a total number of arrangements of word MADHUBANI in such a way that the word is not starting with M but ends with I equal to 17640.

14. Question

Find the number of numbers, greater than a million that can be formed with the digit 2, 3, 0, 3, 4, 2, 3.

Answer

Given, the numbers 2, 3, 0, 3, 4, 2, 3. Total of 7 digits, and it has 2 repeated digits 2, and 3 repeating twice and thrice respectively.

To find: Number of seven-digit numbers that can be formed using digits 2, 3, 0, 3, 4, 2, 3. Notice that an arrangement in which digit 0 in the first place will not be counted as seven-digit number, i.e. greater than 1 Million (1000000). For example- 0232334 will not be counted as seven-digit number since it is a 6 digit number.

The problem can now be rephrased as to find total number of permutations of 7 objects (2, 3, 0, 3, 4, 2, 3) of which two objects are of same type (2, 2), and three objects are of another type (3, 3, 3), And all other objects are distinct. But, 0 cannot be in first place (Condition of seven digit number).

Since we know, Permutation of n objects taking r at a time is ^nP_r,and permutation of n objects taking all at a time is n!

And, we also know Permutation of n objects taking all at a time having p objects of same type, q objects of another type, r objects of another type is $\frac{n!}{p!\times q!\times r!}$. i.e. the, number of repeated objects of same type are in denominator multiplication with factorial.

First, we will find a total number of permutations of these 7 digits and then we will go minus all those permutations in which 0 will come in first place. This will give us exactly number of seven-digit numbers that can be formed by permuting the given digits, i.e. 2, 3, 0, 3, 4, 2, 3.

Total number of permutations $=\frac{7!}{2!\times 3!}$

Number of permutations in which 0 will come at the first place will be equal to (Number of ways we can arrange the remaining digits, i.e. 2, 3, 3, 4, 2, 3 in the remaining six places) $\frac{6!}{2!\times 3!}$

Total number of permutations of given digits forming a seven-digit number$=\frac{7!}{2!\times 3!}-\frac{6!}{2!\times 3!}$

= 360

Hence, total number of permutations of 7 digits (2, 3, 0, 3, 4, 2, 3) forming a 7 digit number is equals to 360.

15. Question

There are three copies each of 4 different books. In how many ways can they be arranged in a shelf?

Answer

Given, that there are 4 different books each having 3 instances, having a total of 12 books.

Lets assume that there are 4 different books B1, B2, B3, and B4. So we have 12 books as -

B1, B1, B1, B2, B2, B2, B3, B3, B3, B4, B4, B4.

To find: Number of arrangements of these 12 books in such a way that all arrangements must be distinct.

The problem can now be rephrased as to find number of permutations of 12 objects in which 3 objects are of one type, 3 objects are of another type, 3 objects are of a third type, and remaining 3 objects are of different type.

Since we know, Permutation of n objects taking r at a time is ${}^nP_r$,and permuation of n objects taking all at a time is n!

And, we also know Permutation of n objects taking all at a time having p objects of same type, q objects of another type, r objects of another type is $\frac{n!}{p!\times q!\times r!}$. i.e. the number of repeated objects of same type are in denominator multiplication with factorial.

The number of permutation of 12 objects with repeating books in factor of 3 $= \frac{12!}{3!\times 3!\times 3!\times 3!}$

= 369600

Hence, total number of permutation of given 12 books will be equals to 369600.

16. Question

How many different arrangements can be made by using all the letters in the word 'MATHEMATICS.' How many of them begin with C? How many of them begin with T?

Answer

Given the word MATHEMATICS. It has 11 letters of which the letters M, A, and T are repeating and all letters are repeated twice.

Since we know, Permutation of n objects taking r at a time is ${}^nP_r$,and permutation of n objects taking all at a time is n!

And, we also know Permutation of n objects taking all at a time having p objects of same type, q objects of another type, r objects of another type is $\frac{n!}{p!\times q!\times r!}$. i.e. the, number of repeated objects of same type are in denominator multiplication with factorial.

Total number of permutations of 11 objects with 3 objects repeating twice $= \frac{11!}{2!\times 2!\times 2!}$

= 4989600

To find the number of words starting with the letter C: This will be equal to permutation of 10 letters (excluded the letter C) where 3 letters (M, A, and T) are repeated twice $= \frac{10!}{2!\times 2!\times 2!}$

= 453600

To find the number of words starting with the letter T: This will be equal to permutation of 10 letters (excluded the letter T) where 2 letters (M, A) are repeated twice, which will be equals to

$\Rightarrow \frac{10!}{2!\times 2!}$

= 907200

Hence, a total number of words permuting the letters of word MATHEMATICS is 4989600. A total number of words starting with the letter C is 453600. A total number of words starting with the letter T equals 907200.

17. Question

A biologist studying the genetic code is interested to know the number of possible arrangements of 12 molecules in a chain. The chain contains 4 different molecules represented by the initials in a chain. The chain contains 4 different molecules represented by the initials A (for Adenine), C (for Cytosine), G(for Guanine) and T (for Thymine) and 3 molecules of each kind. How many different such arrangements are possible?

Answer

Given the molecules' initials A, G, T, and C (All are repeated thrice).

AAAGGGTTTCCC

To find: Number of arrangements of these 12 molecules in such a way that all arrangements must be distinct.

The problem can now be rephrased as to find a number of permutations of 12 objects in which 3 objects are of one type, 3 objects are of another type, 3 objects are of a third type, and remaining 3 objects are of different type.

Since we know, Permutation of n objects taking r at a time is ^nP_r,and permutation of n objects taking all at a time is n!

And, we also know Permutation of n objects taking all at a time having p objects of the same type, q objects of another type, r objects of another type is $\frac{n!}{p!\times q!\times r!}$. i.e. the, number of repeated objects of same type are in denominator multiplication with factorial.

The number of permutation of 12 objects with repeating molecules in the factor of 3 $= \frac{12!}{3!\times 3!\times 3!\times 3!}$

= 369600

Hence, total number of permutation of given 12 molecules will be equals to 369600.

18. Question

In how many ways can 4 red, 3 yellow and 2 green discs be arranged in a row if the discs of the same color are indistinguishable?

Answer

Given, the disc: Red, Yellow, and Green. Total of 9 discs and it has 3 repeated discs Green, Red, and Yellow. The disc Green is repeated twice, and disc Red is repeated 4 times, disc Yellow is repeated thrice.

To find: Number of the arrangement of discs.

The problem can now be rephrased as to find total number of permutations of 9 objects (R, R, R, R, Y, Y, Y, G, G) of which three objects are of same type (Y, Y, Y), and two objects are of another type (G, G), and 4 objects are of different type (R, R, R, R).

Since we know, Permutation of n objects taking r at a time is ^nP_r,and permutation of n objects taking all at a time is n!

And, we also know Permutation of n objects taking all at a time having p objects of the same type, q objects of another type, r objects of another type is $\frac{n!}{p!\times q!\times r!}$. i.e. the, number of repeated objects of same type are in denominator multiplication with factorial.

Total number of such permutations will be $\frac{9!}{4!\times 2!\times 3!}$

$= \frac{5\times 6\times 7\times 8\times 9}{(2\times 1)\times(3\times 2\times 1)}$

= 1260

Hence, total number of permutations of the given type is 1260.

19. Question

How many numbers greater than 1000000 can be formed by using the digits 1, 2, 0, 2, 4, 2, 4?

Answer

Given, the numbers 1, 2, 0, 2, 4, 2, 4. Total of 7 digits, and it has 2 repeated digits 2, and 4 repeating thrice and twice respectively.

To find: Number of seven-digit numbers that can be formed using digits 1, 2, 0, 2, 4, 2, 4. Notice that an arrangement in which digit 0 in the first place will not be counted as seven-digit number, i.e. greater than 1 Million (1000000). For example- 0122442 will not be counted as seven-digit number since it is a 6 digit number.

The problem can now be rephrased as to find total number of permutations of 7 objects (1, 2, 0, 2, 4, 2, 4) of which two objects are of same type (4, 4), and three objects are of another type (2, 2, 2), And all other objects are distinct. But, 0 cannot be in first place (Condition of seven digit number).

Since we know, Permutation of n objects taking r at a time is ^nP_r,and permutation of n objects taking all at a time is n!

And, we also know Permutation of n objects taking all at a time having p objects of the same type, q objects of another type, r objects of another type is $\frac{n!}{p!\times q!\times r!}$. i.e. the, number of repeated objects of same type are in denominator multiplication with factorial.

First, we will find a total number of permutations of these 7 digits and then we will go minus all those permutations in which 0 will come in first place. This will give us exactly number of seven-digit numbers that can be formed by permuting the given digits, i.e. 1, 2, 0, 2, 4, 2, 4.

Total number of permutations $= \frac{7!}{2!\times 3!}$

Number of permutations in which 0 will come at the first place will be equal to (Number of ways we can arrange the remaining digits, i.e. 1, 2, 2, 4, 2, 4 in the remaining six places) $= \frac{6!}{2!\times 3!}$

Total number of permutations of given digits forming a seven-digit number$= \frac{7!}{2!\times 3!} - \frac{6!}{2!\times 3!}$

= 360

Hence, total number of permutations of 7 digits (1, 2, 0, 2, 4, 2, 4) forming a 7 digit number is equals to 360.

20. Question

In how many ways can the letters of the word ASSASSINATION be arranged so that all the S's are together?

Answer

Given, the word ASSASSINATION. It has 13 letters of which repeated letters are A, S, I, and N repeating thrice, 4 times, twice and twice respectively.

We have to find number of words that can formed in such a way that all S's must come together. For example- ASSSSANNIITO, ININAASSSSATO are few words among them. Notice that all S letters comes in bunch (In bold letters).

A specific method is usually used for solving such type of problems. According to that we assume the group of letters that are remains together (here S, S, S, S) are assumed to be a single letter and all other letters are as usual counted as single letter. Now find number of ways as usual; number of ways of arranging r letters from a group of n letters is equals to ^nP_r. And the final answer is then multiplied by number of ways we can arrange the letters in that group which has to be sticked together in it (Here S, S, S, S).

Since we know, Permutation of n objects taking r at a time is ^nP_r,and permutation of n objects taking all at a time is n!

And, we also know Permutation of n objects taking all at a time having p objects of same type, q objects of another type, r objects of another type is $\frac{n!}{p!\times q!\times r!}$. i.e. the number of repeated objects of same type are in denominator multiplication with factorial.

Now,

Letters of word ASSASSINATION: 13

Now from our method, letters are SSSS, A, A, A, I, I, N, N, T, O. (10 letters)

Total number of arrangements of 10 letters $= \frac{10!}{3!\times 2!\times 2!}$

And total number of arrangements of grouped letters (Here S, S, S, S) is $\frac{4!}{4!}$, equals 1.

So, our final answer for arranging the letters such that all vowels sticks together equals multiplication of

$\frac{10!}{3!\times2!\times2!}$ and $\frac{4!}{4!}$

Total number of arrangements $= \frac{10!}{3!\times2!\times2!} \times \frac{4!}{4!}$

= 151200

Hence, total number of words formed during arrangement of letters of word ASSASSINATION such that all S's remains together is equals to 151200.

21. Question

Find the total number of permutations of the letters of the word 'INSTITUTE'.

Answer

Given, the word INSTITUTE. It has 9 letters and it has 2 repeated letters 'I' and 'T'. The letter I is repeated twice, and letter T is repeated thrice. And all other letters are distinct.

The problem can now be rephrased as to find total number of permutations of 9 objects (I, N, S, T, I, T, U, T, E) of which two objects are of same type (I, I), three objects are of another type (T, T, T).

Since we know, Permutation of n objects taking r at a time is ^nP_r,and permutation of n objects taking all at a time is n!

And, we also know Permutation of n objects taking all at a time having p objects of same type, q objects of another type, r objects of another type is $\frac{n!}{p!\times q!\times r!}$. i.e. the number of repeated objects of same type are in denominator multiplication with factorial.

Total number of such permutations $= \frac{9!}{2!\times3!}$

= 30240

Hence, total number of permutations of the word INSTITUTE is 30240.

22. Question

The letters of the word 'SURITI' are written in all possible orders, and these words are written out as in a dictionary. Find the rank of the word 'SURITI'.

Answer

Given the word SURITI.

Arranging the permutations of the letters of the word SURITI in a dictionary:

To find: Rank of word SURITI in dictionary.

First comes, words starting with letter I = 5! = 120

words starting from letter R = 5!/2! = 60

words starting from SI = 4! = 24 (4 letters no repetation)

words starting from SR = 4!/2! = 12 (4 letters, one repetation of I)

words starting from ST = 4!/2! = 12 (4 letters, one repetation of I)

words starting from SUI = 3! = 6 (3 letters no repetation)

words starting from SUR; SURIIT = 1

SURITI = 1

Rank of the word SURITI = 120 + 60 + 24 + 12 + 12 + 6 + 1 + 1

= 236

Hence the rank of the word SURITI in arranging the letters of SURITI in a dictionary among its permutations is 236.

23. Question

If the letters of the word, 'LATE' be permutated and the words so formed be arranged as in a dictionary, find the rank of the word LATE.

Answer

Given the word LATE.

Arranging the permutations of the letters of the word LATE in a dictionary:

To find: Rank of word LATE in dictionary.

First comes, words starting with letter A = 3! (3 letters, no repetation)

words starting with letter E = 3! (3 letters, no repetation)

words starting with L

words starting with LA:

LAET = 1

LATE = 1

Rank of the word LATE = 6 + 6 + 1 + 1

= 14

Hence the rank of the word LATE in arranging the letters of LATE in a dictionary among its permutations is 14.

24. Question

If the letters of the word 'MOTHER' are written in all possible orders and these words are written out as in a dictionary, find the rank of the word 'MOTHER'.

Answer

Given the word MOTHER.

Arranging the permutations of the letters of the word MOTHER in a dictionary:

To find: Rank of word MOTHER in dictionary.

First comes, words starting from letter E = 5! (5 letters no repetation) = 120

words starting from letter H = 5! (5 letters no repetation) = 120

words starting from ME = 4! (4 letters no repetation) = 24

words starting from MH = 4! (4 letters no repetation) = 24

words starting from MOE = 3! (3 letters no repetation) = 6

words starting from MOH = 3! (3 letters no repetation) = 6

words starting from MOR = 3! (3 letters no repetation) = 6

words starting from MOTE = 2! (2 letters no repetation) = 2

words starting from MOTH:

MOTHER = 1

Rank of the word MOTHER = 120 + 120 + 24 + 24 + 6 + 6 + 6 + 2 + 1

= 309

Hence the rank of the word MOTHER in arranging the letters of MOTHER in a dictionary among its permutations is 309.

25. Question

If the permutations of a, b, c, d, e taken all together be written down in alphabetical order as in dictionary and numbered, find the rank of the permutation debac.

Answer

Given the letters a, b, c, d, and e.

Arranging the permutations of the letters a, b, c, d, and e in a dictionary:

To find: Rank of word debac in dictionary.

First comes, words starting from letter a = 4! = 24

words starting from letter b = 4! = 24

words starting from letter c = 4! = 24

words starting from letter d:

words starting from da = 3! = 6

words starting from db = 3! = 6

words starting from dc = 3! = 6

words starting from dea = 2! = 4

words starting from deb:

debac = 1

Rank of the word debac = 24 + 24 + 24 + 6 + 6 + 6 + 4 + 1

= 95

Hence the rank of the word debc in arranging the letters a, b, c, d, and e in a dictionary among its permutations is 95.

26. Question

Find the total number of ways in which six '+' and four '-' signs can be arranged in a line such that no two '-' signs occur together.

Answer

METHOD: 1 (Method of Permutations)

Given 6 '+' and 4 '-' signs to arrange.

To find: Number of arrangements such that no two '-' comes together.

A number of ways of arrangements of 6 '+' signs = 1 (Since all '+' signs are identical).

Arranging all 6 + signs in such a way that now minus sign will only occupy a place in between or edge of + signs.

For example- "P+P+P+P+P+P+P" Arranging 6 + signs gives 7 positions in which a '-' sign can be placed (represented here as P).

Now, number of ways of arrangements of 4 '-' signs in 7 places (Placed all +'s in an alternate manner leading 7 positions) = $^7P_4$

But all 4 '-' signs are identicals hence a number of ways of arrangements of 4 '-' signs among 7 positions = $^7P_4 / 4!$

A total number of arrangments of + and – signs are given by $^7P_4 / 4!$

$= \frac{7!}{3! \times 4!}$

= 5 x 7

= 35

Hence, a total number of arrangements of + and – signs is 35.

METHOD: 2 (Method of combinations)

All 6 + signs can be arranged in 1 way (All are identical).

Now we have 7 places and 4 '-' signs: So we have to select a position and arrange 4 signs among 7 places. In how many ways can we did it? We can do it in $^7C_4$ number of ways which indeed equal to

$$= \frac{7!}{3! \times 4!}$$

= 5 x 7

= 35

Hence, total number of arrangements of + and – signs is 35.

27. Question

In how many ways can the letters of the word "INTERMEDIATE" be arranged so that :

i. the vowels always occupy even places?

ii. the relative order of vowels and consonants do not alter?

Answer

Given the word INTERMEDIATE. IT has 12 words out of which 6 are vowels, and 6 of them are consonants.

(i) the vowels always occupy even places?

i.e., There are 6 vowels, and there can occupy even places, i.e. position number 2, 4, 6, 8, 10, and 12.

A number of ways of arranging 6 vowels among 6 places = 6!/(2! x 3!)

(Since there are 2 repeating vowels I (twice) and E (thrice)).

A number of ways of arranging 6 consonants among 6 places = 6! / 2!

(Since there are 1 repeating consonant T repeating twice).

Total number of ways of arranging vowels and consonants such that vowels can occupy only even positions $= \frac{6!}{2!} \times \frac{6!}{2! \times 3!}$

= 21600

Hence, a number of ways of arranging INTERMEDIATE's letters such that vowels can occupy only even positions is equals to 21600.

(ii) The relative order of vowels and consonants does not alter?

A number of ways of arranging vowels = 6! / (2! x 3!)

A number of ways of arranging consonants = 6! / 2!

Total number of ways of arranging the letter of word INTERMEDIATE such that the relative order of vowels and consonants does not alter $= \frac{6!}{2! \times 3!} \times \frac{6!}{2!}$

= 21600

Hence, a total number of ways of arranging the letters of the word INTERMEDIATE such that the relative orders of vowels and consonants do not change is 21600.

28. Question

The letters of the word 'ZENITH' are written in all possible orders. How many words are possible if all these words are written out as in a dictionary? What is the rank of the word 'ZENITH'?

Answer

Given the word ZENITH. It has 6 letters.

To find: Total number of words that can be generated by relative arranging the letters of the word ZENITH.

Since it has 6 letters with no repetition, therefore the number of ways of arranging 6 letters on 6 positions is 6! = 720

To find: Rank of word ZENITH when all its permutations are arranged in alphabetical order, i.e. in a dictionary.

First comes, the words starting from the letter E = 5! = 120

words starting from the letter H = 5! = 120

words starting from the letter I = 5! = 120

words starting from the letter N = 5! = 120

words starting from the letter T = 5! = 120

words starting from letter Z:

words starting from ZE:

words starting from ZEH = 3! = 6

words starting from ZEI = 3! = 6

words starting from ZEN:

words starting from ZENH = 2! =2

words starting from ZENI:

words starting from ZENIHT = 1

ZENITH = 1

The rank of word ZENITH = 120 + 120 + 120 + 120 + 120 + 6 + 6 + 2 + 1 + 1

= 616

Hence, the rank of the word ZENITH when arranged in the dictionary is 616.

Very Short Answer

1. Question

In how many ways can 4 letters be posted in 5 letter boxes?

Answer

First letter can be posted in any of the 5 letter boxes i.e. in 5 ways.

Similarly, each one of the other 3 letters can be posted in any of the 5 letter boxes i.e. in 5 ways.

As, the operations are dependent, so, total number of ways $= 5 \times 5 \times 5 \times 5$

$= 5^4$.

2. Question

Write the number of 5 digit numbers that can be formed using digits 0, 1 and 2.

Answer

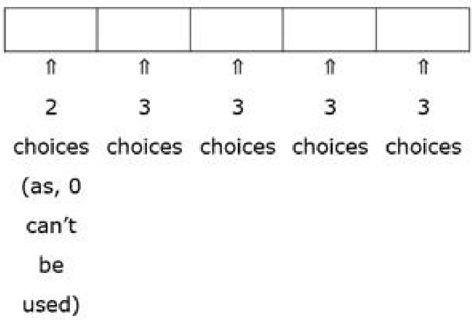
Ten thousands' place (10^4) can be filled with 1 and 2 i.e. ten thousands' place can be filled in 2 ways. [as, it cannot be filled with 0, because it will become a 4-digit number then]

From the thousands' place (10^3) to the ones' (10^0) place can be filled with 0 or 1 or 2, so, 4 places can be filled in 3 ways.

As, the operations are dependent, so, total number of ways $= 2 \times 3 \times 3 \times 3 \times 3$

$= 2 \times 3^4$.

The discussion can be shown pictorially as:



3. Question

In how many ways 4 women draw water from 4 taps, if no tap remains unused?

Answer

1^{st} woman can draw water from anyone of the 4 taps.

Now, as, it is given, that, no tap should remain unused so, 2^{nd} woman can draw water from any of the remaining 3 taps.

Similarly, 3^{rd} women can draw water from any of the remaining 2 taps.

And, 4^{th} woman have to draw water from the remaining 1 tap.

As, the operations are dependent, so, total number of ways $= 4 \times 3 \times 2 \times 1$

= <u>4!</u>

<u>Alternative Approach:</u>

The number of permutations of 4 objects taken 4 at a time is $= {}^4P_4$ = <u>4!</u>.

4. Question

Write the total number of possible outcomes in a throw of 3 dice in which at least one of the dice shows an even number.

Answer

Total possible outcomes in a throw of 3 dice = 6^3 = 216.

When, no die shows an even number i.e. all the die shows odd number (1, 3 or 5), then, the number of possible outcomes = 3^3 = 27.

Hence, the total number of possible outcomes in a throw of 3 dice in which at least one of the dice shows an even number = 216 - 27

= <u>189</u>.

5. Question

Write the number of arrangements of the letters of the word BANANA in which two N's come together.

Answer

As, it is required that two N's come together, so, let us consider {NN} as a single object.

Thus we have, 5 objects {B}, {A}, {NN}, {A}, {A}, and there are 3 A's.

So, the number of arrangements in which the two N's are together is

$= \frac{5!}{3!}$

= 20

6. Question

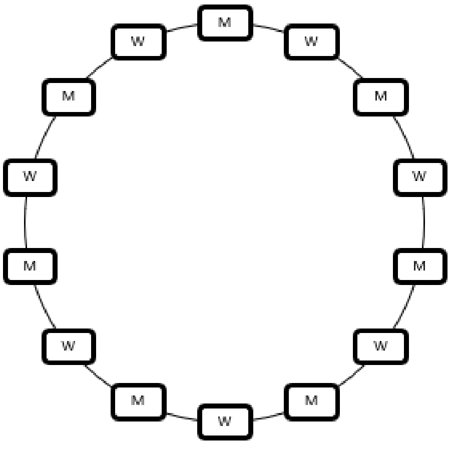
Write the number of ways in which 7 men and 7 women can sit on a round table such that no two women sit together.

Answer

7 men can be arranged in (7-1)! = 6! ways to sit on a round table.

Now, we can place 7 women in 7 empty seats between them so that no two women will be together, and this can be done in 7! ways.

As, the operations are dependent, so, the number of ways in which 7 men and 7 women can sit on a round table such that no two women sit together $=6! \times 7!$



The discussion can be shown pictorially as:

[W = seats between the 7 men(M)]

7. Question

Write the number of words that can be formed out of the letters of the word COMMIT1EE'.

Note: The corrected question may be as follows:

Write the number of words that can be formed out of the letters of the word 'COMMITTEE'.

Answer

We have, 9 objects {C}, {O}, {M}, {M}, {I}, {T}, {T}, {E}, {E} and there are 2 M's, 2 T's, 2 E's.

So, the number of words that can be formed out of the letters of the word 'COMMITTEE' is

$$= \frac{9!}{2! \cdot 2! \cdot 2!}$$

$$= \frac{9!}{(2!)^3}$$

8. Question

Write the number of all possible words that can be formed using the letters of the word 'MATHEMATICS'.

Answer

We have, 11 objects {M}, {A}, {T}, {H}, {E}, {M}, {A}, {T}, {I}, {C}, {S} and there are 2 M's, 2 A's, 2 T's.

So, the number of words that can be formed out of the letters of the word 'MATHEMATICS' is

$$= \frac{11!}{2! \cdot 2! \cdot 2!}$$

9. Question

Write the number of ways in which 6 men and 5 women can dine at a round table if no two women sit together.

Answer

6 men can be arranged in (6-1)! = 5! ways to dine at a round table.

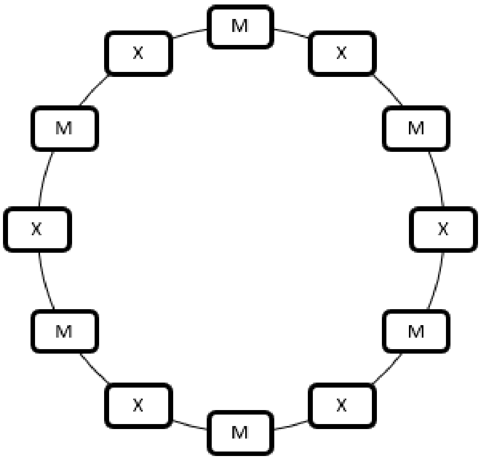
Now, if we place 5 women in 6 empty seats between them so that no two women will be together, and this can be done in $^6P_5$ ways i.e. in

$$\frac{6!}{(6-5)!} = 6! \text{ ways}$$

As, the operations are dependent, so, the number of ways in which 6 men and 5 women can dine at a round table if no two women sit together.

=5!×6!

The discussion can be shown pictorially as:



[X = 6 empty seats between the 6 men(M)]

10. Question

Write the number of ways in which 5 boys and 3 girls can be seated in a row so that each girl is between 2 boys.

Answer

5 boys can be seated in 5! ways to sit in a row. Now, we can place 3 girls in 4 empty seats between them so that each girl is between 2 boys, and this can be done in $^4P_3$ = 24 ways.

As, the operations are dependent, so, the number of ways in which 5 boys and 3 girls can be seated in a row so that each girl is between 2 boys

$= 5! \times 24$ = <u>2880</u>.

The discussion can be shown pictorially as:

B	X	B	X	B	X	B	X	B

[X = 4 empty seats between 2 boys(B)]

11. Question

Write the remainder obtained when 1! + 2! + 3! + ...+ 200! is divided by 14.

Answer

We can see, from 7! onwards the terms are divisible by 14.

∵7!=7×6×5×4×3×2×1

=(7×2)×6×5×4×3

$=14\times360$

So, we should consider up to 6! term to obtain the remainder.

Now,

1! + 2! + 3! + 4! + 5! + 6!

= 1 + 2 + 6 + 24 + 120 + 720

= 873

On dividing 873 by 14, we get 5 as remainder.

$\because 14\times62=868$

and 873-868=5

12. Question

Write the number of numbers that can be formed using all for digits 1,2,3,4.

Answer

Thousands' place (10^3) can be filled with 1, 2, 3 and 4 i.e. thousands' place can be filled in 4 ways.

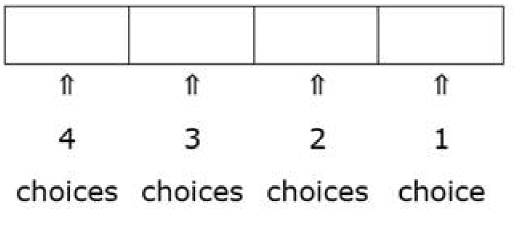
Hundreds' place (10^2) can be filled in 3 ways [i.e. with the remaining 3 digits].

Similarly, tens' place (10^1) can be filled in 2 ways and ones' place can be filled only in 1 way.

As, the operations are dependent, so, total number of ways $= 4 \times 3 \times 2 \times 1$

=24

The discussion can be shown pictorially as:



ALTERNATIVE APPROACH:

The number of permutations of 4 objects taken 4 at a time is $= {}^4P_4 = 4! = 24$.

MCQ

1. Question

Mark the correct alternative in the following:

The number of permutations of n different things taking r at a time when 3 particular things are to be included is

A. ${}^{n-3}P_{r-3}$

B. ${}^{n-3}P_r$

C. ${}^nP_{r-3}$

D. $r!\,{}^{n-3}C_{r-3}$

Answer

Primarily, excluding the 3 things which are to be included, we have to select (r - 3) things from (n - 3), this can be done in ${}^{n-3}C_{r-3}$ ways.

Now, the 3 particular things can be selected from 3 remaining things only in 1 way.

And the selected r things can be arranged in r! ways.

So, the number of permutations of n different things taking r at a time when 3 particular things are to be included is = $r!\ {}^{n-3}C_{r-3}$

2. Question

Mark the correct alternative in the following:

The number of five-digit telephone numbers having at least one of their digits repeated is

A. 90000

B. 100000

C. 30240

D. 69760

Answer

Total number of five-digit telephone numbers = 10^5 = 100000.

When no digit is repeated, then the number of five-digit telephone numbers = ${}^{10}P_5$

[as, 5 digits among 10 digits can be arranged in ${}^{10}P_5$ ways]

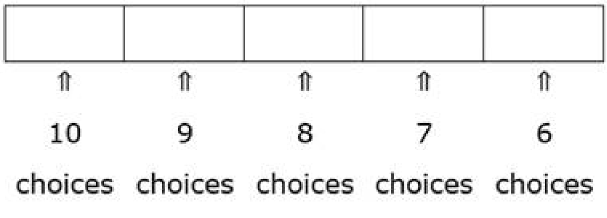
$=10\times9\times8\times7\times6$

= 30240

So, the number of five-digit telephone numbers having at least one of their digits repeated is = 100000 – 30240

= 69760.

The discussion can be shown pictorially as:



3. Question

Mark the correct alternative in the following:

The number of words that can be formed out of the letters of the word “ARTICLE” so that vowels occupy even places is

A. 574

B. 36

C. 754

D. 144

Answer

1 2 3 4 5 6 7

We have, 7 objects {A}, {R}, {T}, {I}, {C}, {L}, {E} and there are 3 vowels {A}, {E}, {I}; and the vowels should occupy even places i.e. 2, 4 and 6^{th} place.

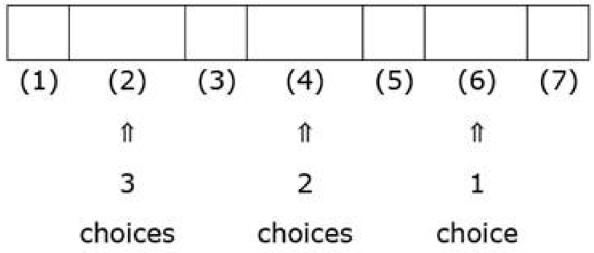
Now, 3 vowels can be placed in 3 places in 3! ways.

And, the other alphabets i.e. the 4 consonants can be placed in 4 places in 4! ways.

So, the number of words that can be formed out of the letters of the word "ARTICLE" so that vowels occupy even places is $= 3! \times 4!$

$= 6 \times 24$

= 144.



4. Question

Mark the correct alternative in the following:

How many numbers greater than 10 lacs be formed from 2, 3, 0, 3, 4, 2, 3?

A. 420

B. 360

C. 400

D. 300

Answer

We know, 10 lacs = 10,00,000 and it has 7 places.

The given numbers are 2, 3, 0, 3, 4, 2, 3.

As the required number is greater than 10 lacs, so the 7^{th} place can be filled with all the digits except 0.

So, the place can be filled in 6 ways.

The other 6 places can be filled with other 6 digits, so the total number of ways to fill the remaining 6 places is = 6!

But 2 is repeated twice and 3 is repeated thrice.

So, total numbers greater than 10 lacs be formed from 2, 3, 0, 3, 4, 2, 3 is

$= \frac{6 \times 6!}{2! \times 3!}$

$= \frac{6 \times 720}{2 \times 6}$

=360.

5. Question

Mark the correct alternative in the following:

The number of different signals which can be given from 6 flags of different colours taking one or more at a time, is

A. 1958

B. 1956

C. 16

D. 64

Answer

Number of signals which can be given using 1 flag = ${}^6P_1$ = 6

Number of signals which can be given using 2 flag = ${}^6P_2$ = 30

Number of signals which can be given using 3 flag = ${}^6P_3$ = 120

Number of signals which can be given using 4 flag = ${}^6P_4$ = 360

Number of signals which can be given using 5 flag = ${}^6P_5$ = 720

Number of signals which can be given using 6 flag = ${}^6P_6$ = 720

As, the operations are independent, so, the total number of different signals which can be given from 6 flags of different colours taking one or more at a time, is

= 6 + 30 + 120 + 360 + 720 + 720

=1956.

6. Question

Mark the correct alternative in the following:

The number of words from the letters of the word 'BHARAT' in which B and H will never come together, is

A. 360

B. 240

C. 120

D. none of these

Answer

We have, 6 objects {B}, {H}, {A}, {R}, {A}, {T} and there are 2 A's.

So, the number of words that can be formed out of the letters of the word 'BHARAT' is

$= \frac{6!}{2!}$

= 360

The number of words that can be formed out of the letters of the word 'BHARAT' is in which {B} and {H} come together is = 5!

= 120.

[considering {BH} as a single object]

Hence, the number of words from the letters of the word 'BHARAT' in which B and H will never come together, is = (360 – 120)

= 240.

7. Question

Mark the correct alternative in the following:

The number of six letter words that can be formed using the letters of the word "ASSIST" in which S's alternate with other letters is

A. 12

B. 24

C. 18

D. none of these

Answer

We have, 6 objects {A}, {S}, {S}, {I}, {S}, {T} and there are 3 S's.

The S’s can be arranged in following two ways to form a six letter word in which the S’s alternate with other letters, is as shown below:

S×S×S× and ×S×S×S where the ‘×’ are to be filled with the other letters.

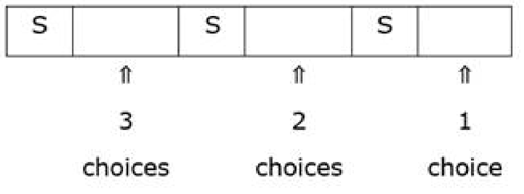
Now, for the other 3 positions we have to place A, I and T, so, 3 positions can be filled with 3 letters in 3!

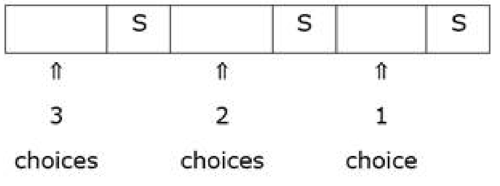
= 6 ways.

The number of six letter words that can be formed using the letters of the word “ASSIST” in which S’s alternate with other letters is $= 2 \times 6$

= <u>12</u>.

The discussion can be shown pictorially as:





8. Question

Mark the correct alternative in the following:

The number of arrangements of the word “DELHI” in which E precedes I is

A. 30

B. 60

C. 120

D. 59

Answer

Detailed Solution

Considering {E} and {I} as single object, we have {D}, {EI}, {L}, {H}.

The number of words that can be formed out of the letters of the word “DELHI” in which {E} precedes {I} is = 4!

= 24. (When {E} is just before {I})

However, it is not required to place {E} just before {I}, so, in that case, the solution will be:

<u>Case-1:</u> Let us say, the first letter of the word be {E}, so the word format becomes E××××, where the ‘×’ are to be filled with the other 4 letters, this can be done in 4! = 24 ways.

The discussion can be shown pictorially as:

E				
	⇑	⇑	⇑	⇑
	4	3	2	1
	choices	choices	choices	choice

Case-2: Let us say, the second letter of the word be {E}, so the word format becomes ×E×××, where the first '×' is to be filled with anyone of the 3 letters i.e. {D}, {L}, {H} [not with {I}], this can be done in 3 ways.

And, the other 3 '×' are to be filled with the remaining 3 letters, this can be done in 3! = 6 ways.

So, in this case total number of arrangements

$= 3 \times 6$

= 18.

The discussion can be shown pictorially as:

	E			
⇑		⇑	⇑	⇑
3		3	2	1
choices (except "I")		choices	choices	choice

Case-3: Let us say, the third letter of the word be {E}, so the word format becomes ××E××, where the first 2 '×' are to be filled with any 2 of 3 letters i.e. {D}, {L}, {H} [not with {I}], this can be done in 3×2 = 6 ways.

And, the other 2 '×' are to be filled with the remaining 2 letters, this can be done in 2! = 2 ways.

So, in this case total number of arrangements

$= 6 \times 2$

= 12.

The discussion can be shown pictorially as:

		E		
⇑	⇑		⇑	⇑
3	2		2	1
choices (except "I")	choices (except "I")		choices	choice

Case-4: Let us say, the third letter of the word be {E}, so the word format becomes ×××E×, where the first 3 '×' are to be filled with 3 letters i.e. {D}, {L}, {H} [not with {I}], this can be done in 3! = 6 ways.

And, the last '×' is to be filled with the remaining {I}, this can be done only in 1 way.

So, in this case total number of arrangements

$= 6 \times 1$

= 6.

The discussion can be shown pictorially as:

			E	
⇑	⇑	⇑		⇑
3 choices (except "I")	2 choices (except "I")	1 choice (except "I")		1 choice "I"

As, the cases are independent, so, the total number of arrangements of the word "DELHI" in which E precedes I is = 24 + 18 + 12 + 6

=60.

ALTERNATIVE METHOD

We have, 5 objects {D}, {E}, {L}, {H}, {I}.

So, the number of words that can be formed out of the letters of the word "DELHI" is = 5!

= 120.

From, symmetry, we can say, out of these 120 words in 60 words {E} precedes {I} and in other 60 words {I} precedes {E}.

So the required answer = 60.

9. Question

Mark the correct alternative in the following:

The number of ways in which the letters of the word 'CONSTANT' can be arranged without changing the relative positions of the vowels and consonants is

A. 360

B. 256

C. 444

D. none of these

Answer

We have, 8 objects {C}, {O}, {N}, {S}, {T}, {A}, {N}, {T} and there are 2 N's, 2 T's.

As, we are concerned about the relative positions of the vowels and consonants, so, by denoting the consonants by "×" and the vowels by "_", we get the structure of the word as, ×_×××_××

Where, the "×" are to be filled with 6 consonants and this can be done in

$$= \frac{6!}{2! \times 2!}$$

= 180 ways. [as, there are 2 N's, 2 T's]

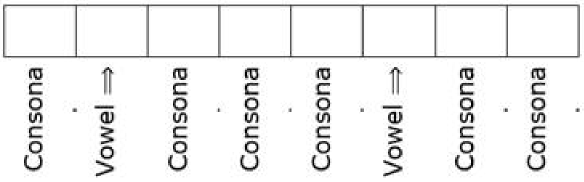
And the "_" are to be filled with 2 vowels, and this can be done in 2! = 2 ways.

As, the cases are independent, so, the number of ways in which the letters of the word 'CONSTANT' can be arranged without changing the relative positions of the vowels and consonants is

$= 180 \times 2$

= 360.

The discussion can be shown pictorially as:



10. Question

Mark the correct alternative in the following:

The number of ways to arrange the letters of the word CHEESE are

A. 120

B. 240

C. 720

D. 6

Answer

We have, 6 objects {C}, {H}, {E}, {E}, {S}, {E} and there are 3 E's.

So, the number of words that can be formed out of the letters of the word ‘CHEESE’ is

$= \frac{6!}{3!}$

= 120.

11. Question

Mark the correct alternative in the following:

Number of all four digit numbers having different digits formed of the digits 1, 2, 3, 4 and 5 and divisible by 4 is

A. 24

B. 30

C. 125

D. 100

Answer

We know, any number to be divisible by 4, we must have the last 2 digits of the number divisible by 4.

So, the whole four-digit number will be divisible by 4 if the last 2 digits are: 12, 32, 52, 24.

Case-1: When, we have 2 in ones’ place:

In this case, we can fill tens’ place with 1 ,3 or 5 i.e. we have 3 ways to fill the tens’ place.

Hundreds’ place can be filled with any 2 of the remaining 3 digits [2 digits are used to fill the ones’ and tens’ place among the given 5 digits], in $^3P_2$ = 6 ways.

So, number of four-digit numbers that can be formed in this case $= 3 \times 6$

= 18.

Case-2: When, we have 4 in ones’ place:

In this case, we can fill tens’ place with 2 i.e. tens’ place can be filled only in 1 way.

Hundreds’ place can be filled with any 2 of the remaining 3 digits [2 digits are used to fill the ones’ and tens’ place among the given 5 digits], in $^3P_2$ = 6 ways.

So, number of four-digit numbers that can be formed in this case = 6.

As, the two cases are independent so, the total number of four-digit numbers that can be formed of the digits 1, 2, 3, 4 and 5 and divisible by 4 is = 18 + 6

= 24.

12. Question

Mark the correct alternative in the following:

If the letters of the word KRISNA are arranged in all possible ways and these words are written out as in a dictionary, then the rank of the word KRISNA is

A. 324

B. 341

C. 359

D. none of these

Answer

We have, 6 objects {K}, {R}, {I}, {S}, {N}, {A}.

So, the total number of words that can be formed using the letters of the word “KRISNA” = 6!

= 720.

Writing the letters of the word “KRISNA” alphabetically, i.e. {A}, {I}, {K}, {N}, {R}, {S}.

For “KRISNA” word starting with	Word starting with	Number of words
K	A	5! = 120
	I	5! = 120
KR	KA	4! = 24
	KI	4! = 24
	KN	4! = 24
KRI	KRA	3! = 6
KRIS	KRIA	2!=2
	KRIN	2!=2
KRISN	KRISA	1! = 1
	Total number of words before “KRISNA”	323

Hence, there are 323 words before “KRISNA”, then the rank of the word “KRISNA” is (323 + 1)

= 324.

13. Question

Mark the correct alternative in the following:

If in a group of n distinct objects, the number of arrangements of 4 objects is 12 times the number of arrangements of 2 objects, then the number of objects is

A. 10

B. 8

C. 6

D. none of these

Answer

In a group of n distinct objects, the number of arrangements of 4 objects is = ${}^{n}P_4$ and the number of arrangements of 2 objects is = ${}^{n}P_2$.

According to the given problem,

$P_4^n = 12 \times P_2^n$

$\Rightarrow \frac{n!}{(n-4)!} = 12 \times \frac{n!}{(n-2)!}$

$\Rightarrow n \times (n-1) \times (n-2) \times (n-3) = 12 \times n \times (n-1)$

$\Rightarrow (n-2) \times (n-3) = 12 = (6-2)(6-3)$

$\Rightarrow n = 6$

Hence, the number of objects = 6.

[Note: $P_4^n = {}^{n}P_4$ and $P_2^n = {}^{n}P_2$]

14. Question

Mark the correct alternative in the following:

The number of ways in which 6 men can be arranged in a row so that three particular men are Consecutive, is

A. 4! × 3!

B. 4!

C. 3! × 3!

D. none of these

Answer

As, it is required that three particular men are consecutive, so, let us consider the three particular men as a single object.

Thus we have, 4 objects, which can be arranged in 4! ways, and the 3 particular men who are consecutive can be arranged in 3! ways among themselves.

So, the number of arrangements in which 6 men can be arranged in a row so that three particular men are consecutive, is

$= 4! \times 3!$.

15. Question

Mark the correct alternative in the following:

A 5-digit number divisible by 3 is to be formed using the digits 0, 1, 2, 3, 4 and 5 without repetition. The total number of ways in which this can be done is

A. 216

B. 600

C. 240

D. 3125

Answer

5-digit number divisible by 3 can be formed using 0, 1, 2, 4, 5 and 1, 2, 3, 4, 5.

When, the 5-digit number is formed using the digits 0, 1, 2, 4 and 5

⇑	⇑	⇑	⇑	⇑
4	4	3	2	1
choices (as, 0 can't be used)	choices	choices	choices	choice

So, the number of ways in which 5-digit number divisible by 3 can be formed using 0, 1, 2, 4, 5 can be done is $= 4 \times 4 \times 3 \times 2 \times 1$

= 96.

So, the number of ways in which 5-digit number divisible by 3 can be formed using 1, 2, 4, 5 can be done is $= {}^5P_5$

$= \frac{5!}{(5-5)!}$

$= \frac{5!}{0!}$

= 120.

The total number of ways in which 5-digit number divisible by 3 can be formed using 0, 1, 2, 3, 4, 5 can be done is = 96 + 120

= <u>216</u>.

16. Question

Mark the correct alternative in the following:

The product of r consecutive positive integers is divisible by

A. r!

B. r! + 1

C. (r + 1)!

D. none of these

Answer

The product of r consecutive positive integers is

=1•2•3•4•...•(r-4)•(r-3)•(r-2)•(r-1)•r

= r!

So, the product of r consecutive positive integers is divisible by <u>r!</u>.

17. Question

Mark the correct alternative in the following:

If ${}^{k+5}P_{k+1} = \frac{11(k-1)}{2} . {}^{k+3}P_{k,}$ then the values of k are

A. 7 and 11

B. 6 and 7

C. 2 and 11

D. 2 and 6

Answer

$P_{k+1}^{k+5} = \frac{11(k-1)}{2} \cdot P_k^{k+3}$

$\Rightarrow \frac{(k+5)!}{((k+5)-(k+1))!} = \frac{11(k-1)}{2} \cdot \frac{(k+3)!}{((k+3)-k)!}$

$\Rightarrow \frac{(k+5)!}{4!} = \frac{11(k-1)}{2} \cdot \frac{(k+3)!}{3!}$

$\Rightarrow \frac{(k+5)(k+4)(k+3)!}{4!} = \frac{11(k-1)}{2} \cdot \frac{(k+3)!}{3!}$

$\Rightarrow \frac{(k+5)(k+4)}{4!} = \frac{11(k-1)}{2} \cdot \frac{1}{3!}$

$\Rightarrow \frac{(k+5)(k+4)}{24} = \frac{11(k-1)}{2} \cdot \frac{1}{6}$

$\Rightarrow k^2 + 9k + 20 = 22(k-1)$

$\Rightarrow k^2 + 9k - 22k + 20 + 22 = 0$

$\Rightarrow k^2 - 13k + 42 = 0$

$\Rightarrow k^2 - 6k - 7k + 42 = 0$

$\Rightarrow k(k-6) - 7(k-6) = 0$

$\Rightarrow (k-6)(k-7) = 0$

$\therefore k = 6$ and 7.

18. Question

Mark the correct alternative in the following:

The number of arrangements of the letters of the word BHARAT taking 3 at a time is

A. 72

B. 120

C. 14

D. none of these.

Answer

We have, 6 objects {B}, {H}, {A}, {R}, {A}, {T} and there are 2 A's.

So, the words can be formed out of the letters of the word 'BHARAT' taking 3 at a time can be done in 2 ways:

Case-1: When all the letters are distinct.

We have, 5 distinct letters, out of which taking three at a time, the number of words that can be formed = ${}^5P_3$

$= \frac{5!}{(5-3)!}$

$= \frac{5!}{2!}$

= 60

Case-2: When 2 A's are selected.

So, we have, 2A's and 1 letter is to selected out of the 4 distinct letters, which can be done in = $^4P_1$

$= \frac{4!}{(4-1)!}$

$= \frac{4!}{3!}$

= 4 ways.

Now, the 3 letters can be arranged among themselves, but there are 2 A's, so the number of ways in which arrangement can be done is

$= \frac{3!}{2!}$

= 3

So, in this case, total number of words that can be formed $= 4 \times 3$

= 12.

The number of arrangements of the letters of the word BHARAT taking 3 at a time is = (60 + 12)

= 72.

19. Question

Mark the correct alternative in the following:

The number of words that can be made by re-arranging the letters of the word APURBA so that vowels and consonants are alternate is

A. 18

B. 35

C. 36

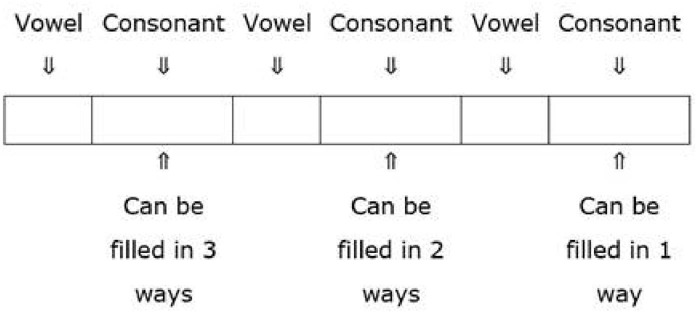
D. none of these

Answer

We have, 6 objects {A}, {P}, {U}, {R}, {B}, {A} and there are 2 A's, so, there are 3 vowels (2 {A}'s and 1 {U}) and 3 consonants ({P}, {R}, {B}).

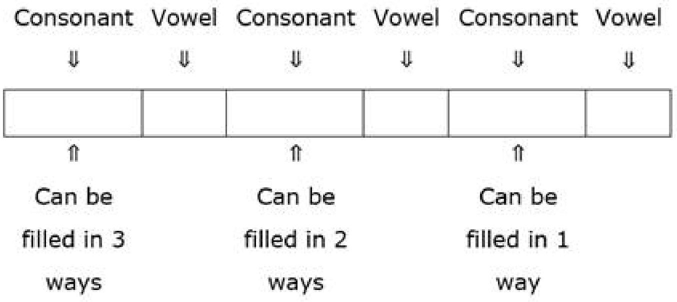
The word arrangement where, vowels and consonants are alternate is shown below:

Arrangement-1:



Arrangement-2:

Arrangement-2:



The vowel positions can be filled in $\frac{3!}{2!} = 3$ ways, and the consonant positions can be filled in 3! ways.

As, the operations are dependent so, the number of ways to fill the 6 positions $= 3 \times 3!$

$= 3 \times 6$

$= 18$

So, the number of words that can be made by re-arranging the letters of the word APURBA so that vowels and consonants are alternate is $= 18 \times 2$

[as, we have 2 possible arrangements]

$= \underline{36}$.

20. Question

Mark the correct alternative in the following:

The number of different ways in which 8 persons can stand in a row so that between two particular persons A and B there are always two persons, is

A. $60 \times 5!$

B. $15 \times 4! \times 5!$

C. $4! \times 5!$

D. none of these

Answer

As, it is required that, two particular persons A and B there are always two persons so, let us consider this arrangement be “A××B” and consider it as a single object.

So, we are left with, 4 persons and an object, i.e. total 5 objects.

Now, this 5 objects can be arranged in 5! ways.

Again, the two ‘×’ are to be filled with 2 persons from 6 persons, this can be done in ${}^6P_2 = 30$ ways.

Two persons ‘A’ and ‘B’ can be arranged in 2! = 2 ways.

So, the total number of different ways in which 8 persons can stand in a row so that between two particular persons A and B there are always two persons, is $= 5! \times 30 \times 2$

$= 5! \times 60$.

21. Question

Mark the correct alternative in the following:

The number of ways in which the letters of the word ARTICLE can be arranged so that even places are always occupied by consonants is

A. 576

B. ${}^4C_3 \times 4!$

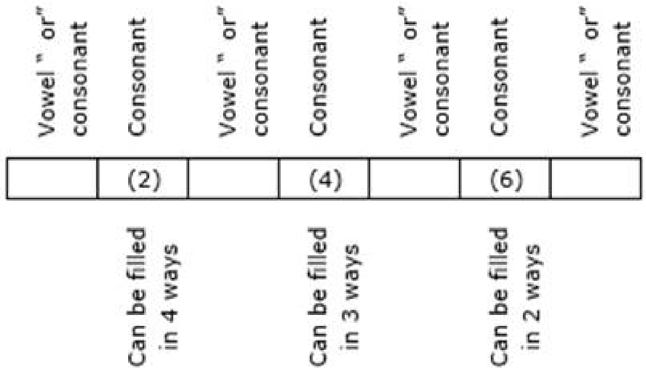
C. $2 \times 4!$

D. none of these

Answer

We have, 7 objects {A}, {R}, {T}, {I}, {C}, {L}, {E} and there are 4 consonants {R}, {T}, {C}, {L}; and the 3 out of 4 of the consonants should occupy even places i.e. 2, 4 and 6^{th} place, this can be done in $4 \times 3 \times 2$

= 24 ways.



And, the other 4 alphabets i.e. the remaining 1 consonant and 3 vowels is to be placed in 4 places in $^4P_4 = 4!$ = 24 ways.

So, the number The number of ways in which the letters of the word ARTICLE can be arranged so that even places are always occupied by consonants is $= 24 \times 24$

= 576.

22. Question

Mark the correct alternative in the following:

In a room there are 12 bulbs of the same wattage, each having a separate switch. The number of ways to light the room with different amounts of illumination is

A. $12^2 - 1$

B. 2^{12}

C. $2^{12} - 1$

D. none of these

Answer

In the room there are 12 bulbs of same wattage, each having separate switch.

We have 2 options for each bulb i.e. either switching it ON or to switching it OFF.

The 1^{st} bulb can be switched ON or OFF i.e. 2 choices.

The 2^{nd} bulb can be switched ON or OFF i.e. 2 choices, and so on.

Now, for 12 bulbs we have total 2^{12} choices.

But, as we have to illuminate the room, so the single choice in which all the 12 bulbs are OFF should be omitted.

So, we have total $(2^{12} - 1)$ ways to light the room with different amounts of illumination.